HANKEL DETERMINANTS WHOSE ELEMENTS ARE SECTIONS OF A TAYLOR SERIES. PART II.

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1. **Introduction.** This paper contains a generalization of a theorem on the zeros of the sections of a Taylor series due to A. Hurwitz [2]. In a previous paper [6], which we refer to as Part I, a generalization was made of the theorem on zeros of sections of a Taylor series due to R. Jentzsch [4]. The method of this paper is an extension of that used in Part I, but can be read independently. In comparing the results of these papers, however, account should be taken of the change of notation introduced in display (4).

In Part I an application was made of formulas due to J. Hadamard [1] for the moduli of certain poles of an analytic function. For the proof of the theorem of this paper, an extension of one of these formulas is required. This result is stated and proved in §3, and then applied to complete the proof in §4.

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2. **Statement of the theorem.** The necessary notations and definitions are first introduced.

2.1 Let $f(z)$ designate an analytic function which has a branch represented in a neighborhood of $z = \infty$ by a series of negative powers of z,

$$f(z) = \sum_{k=0}^{\infty} a_k z^{-(k+1)} \qquad (|\, z \,| > R,\ 0 \leq R < \infty), \tag{1}$$

where R is the radius of convergence of the series. Denote by $s_n(z)$ the n-th section of the power series,

$$s_n(z) = \sum_{k=0}^{n-1} a_k z^{-(k+1)}. \tag{2}$$

A determinant of order k with element b_{ij} in the i-th row and j-th column will be denoted by

$$|\, b_{ij} \,|_1^k . \tag{3}$$

When the element b_{ij} depends only on $i + j$, this is a Hankel determinant. The results of Part I and this paper concern the zeros of the Hankel determinants

$$S_{n,p}(z) = |\, s_{n+i+j-2}(z) \,|_1^p , \tag{4}$$

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