TRANSCENDENTAL NUMBERS OVER CERTAIN FUNCTION FIELDS

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1. Introduction. This paper is concerned with certain transcendental numbers over function fields. The first part (§3) contains a proof of an analogue of a theorem due to G. Faber [1; 552]. It is shown that certain entire functions $f(t)$ have the property that $f(\alpha)$ is transcendental for all algebraic, non-zero α. An interesting corollary to the theorem is that the non-vanishing zeros of $f(t)$ are also transcendental.

The second part (§§4–6) contains proofs of several theorems over fields of characteristic p, all of which are generalizations of or suggested by certain theorems due to L. I. Wade [3], [5]. A typical example of such a theorem is: The series $\sum_{k=0}^{\infty} G_k^{-e_k q}$ is transcendental, where the G_k are polynomials such that $G_k \mid G_{k+1}$; deg $G_0 \geq 1$; $e_0 < e_1 < \cdots$; $q \geq 2$; and $p \nmid e_k$.

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2. Preliminaries. Let F be a field of arbitrary characteristic. Let $\Phi = F[x_1, \cdots, x_w]$ be the domain of polynomials in the indeterminates $x_1, \cdots, x_w$ with coefficients in F. Let $F(x_1, \cdots, x_w)$ denote the quotient field. For $G \varepsilon \Phi$, $g = \deg G$ means total degree of G; put $| G | = k^g$, $k > 1$, $| G_1/G_2 | = | G_1 |/| G_2 |$, thus defining a non-Archimedean valuation. Let Φ^* denote the completion of $F(x_1, \cdots, x_w)$ relative to this valuation.

In §3 the characteristic will be arbitrary, but in the remainder of the paper (§§4–6) the characteristic will be a prime p, and the field F will be the finite field $GF(p^n)$.

By "transcendental" will be meant transcendental relative to $F(x_1, \cdots, x_w)$, similarly, for "algebraic".

3. A proof of an analogue of the Faber Theorem.

Theorem 1. *If*

$$f(t) = \sum_{i=0}^{\infty} C_i t^i \qquad (C_i \varepsilon F(x_1, \cdots, x_w)) \tag{3.1}$$

converges for all $t \varepsilon \Phi^*$, G_n *denotes the least common multiple of the denominators of* $C_0, C_1, \cdots, C_n$, *and there exist infinitely many numbers* $n = n_1, n_2, \cdots$ *such that*

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