CONTINUA AND THEIR COMPLEMENTARY DOMAINS IN THE PLANE

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The results in this paper were obtained as a consequence of considering some questions which were raised by Professor R. L. Moore during the fall of 1949 and spring of 1950.

Notation. The set of all points in the plane is denoted by S. If M is a point set, $\mathrm{cl}(M)$ denotes M together with all of its limit points. If G is a collection of point sets, G^* denotes the sum of the point sets of G.

P. M. Swingle has proved that *if the continuum M is not the sum of three continua such that no one of them is a subset of the sum of the other two and H and K are proper subcontinua of M such that $H + K = M$, then $\mathrm{cl}(M - H)$ and $\mathrm{cl}(M - K)$ are indecomposable continua and $\mathrm{cl}(M - H) + \mathrm{cl}(M - K) = M$.* (See [6; Theorem 2] and the argument thereof.)

Theorem 1. *If the hypothesis of the above theorem is satisfied and L and N are indecomposable proper subcontinua of M such that $L + N = M$, then the continua L and N are the continua $\mathrm{cl}(M - H)$ and $\mathrm{cl}(M - K)$.*

Lemma 1.1. *If M is a continuum, H and K are proper subcontinua of M such that $H + K = M$, and L and N are indecomposable proper subcontinua of M such that $L + N = M$, then neither of the continua H and K is a proper subset of one of the continua L and N.*

Proof of Lemma 1.1. Suppose that H is a proper subset of L. Since L is indecomposable, then $\mathrm{cl}(L - H) = L$. Since K contains $L - H$, then it contains L. But H is a subset of L and not of K. This is a contradiction.

Lemma 1.2. *If the continuum M is not the sum of three continua such that no one of them is a subset of the sum of the other two and H and K are indecomposable proper subcontinua of M such that $H + K = M$, then $H = \mathrm{cl}(M - K)$.*

Proof of Lemma 1.2. Since H contains $M - K$, then $\mathrm{cl}(M - K)$ is a subset of H. By a theorem of Swingle [6; Theorem 1], $\mathrm{cl}(M - K)$ is a continuum. Since $\mathrm{cl}(M - K) + K = M$, then by Lemma 1.1, $\mathrm{cl}(M - K)$ is not a proper subset of H. Therefore $\mathrm{cl}(M - K) = H$.

Proof of Theorem 1. Let $H' = \mathrm{cl}(M - K)$, and let $K' = \mathrm{cl}(M - H)$. Now suppose that the continua L and N are not the continua H' and K'. By Lemma 1.2, $L = \mathrm{cl}(M - N)$, $N = \mathrm{cl}(M - L)$, $H' = \mathrm{cl}(M - K')$ and $K' = \mathrm{cl}(M - H')$.

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