A SIMPLIFICATION OF GAMES IN EXTENSIVE FORM

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When a zero-sum two-person game in extensive form is normalized, it frequently happens that many of the rows and columns of its matrix are repeated. When this is so, one can greatly simplify the problem of finding the value of the game, and optimum strategies for the two players, by crossing out repetitions of rows and columns. Sometimes, however, the number of repetitions is so great, that it is not feasible even to write out the original matrix at all. In such a case, it becomes desirable to transform the given game in extensive form, so as to reduce the number of repetitions in its matrix. In this paper we develop a method of doing this for a certain class of games in extensive form. The method consists essentially in decreasing the number of strategies available to the players, by the elimination of useless information.

Henceforth by a *game* we shall mean a finite zero-sum two-person game which satisfies the following conditions: (1) there is a fixed number, n, of moves, which is the same for all plays of the game; (2) none of the moves are chance moves; (3) the i-th move, for $i = 1, \cdots, n$, is always made by the same player, independent of the past course of the play; (4) at the i-th move, for $i = 1, \cdots, n$, the player making the move chooses an element from a fixed finite set $A(i)$, which does not depend on the past course of the play; (5) the player making the i-th move, for $i = 1, \cdots, n$, knows either everything or nothing about each of the past moves (that is, for each $j < i$, he either knows exactly which element was chosen in the j-th move, or else he knows nothing at all about the choice made at the j-th move—he does not, for example, know merely that the choice was made from a certain two-element proper subset B of $A(i)$).

These restrictions may seem severe, but it should be noticed that many games which do not comply with them can easily be transformed into games which do. Thus a game can be made to satisfy condition (1) by putting in vacuous moves (moves in which the player has only one alternative available, or moves at which all his alternatives lead to the same outcome). And similarly a game can be made to satisfy condition (5) by putting extra moves into the middle of the game. Such processes as these, however, are artificial and are not always even possible in the case of games which fail to satisfy condition (3). It would therefore be desirable to extend our results directly to a wider class of games.

We now define in a more formal way the class of games with which we shall be concerned. We shall use standard set theoretical notations. The set whose only members are $a_1, a_2, \cdots, a_r$ will be indicated by

$$\{a_1, a_2, \cdots, a_r\}.$$

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