DIFFERENTIAL INVARIANTS OF RULED SURFACES BELONGING TO ONE SPECIAL LINEAR COMPLEX

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1. **Introduction.** Professor Vaclav Hlavaty has defined a system of differential invariants of a ruled surface in terms of a set of six complexes which satisfy a particular condition. The set of complexes is defined for a surface not belonging to a constant linear complex, and it is pointed out how an equivalent set may be obtained if the surface belongs to one, two, or three general complexes. It is the purpose of this paper to extend the discussion to surfaces which belong to one special complex, that is, surfaces which contain a line not in the family of rulings, and to present necessary and sufficient conditions that a ruled surface have this character.

The methods used are basically those of the first hundred pages of Hlavaty's *Differentielle Liniengeometrie.* All references are to this book, and where the contrary is not expressly indicated, all symbols and terms have the meanings there given them.

2. **Terminology.** It is assumed that the Pluecker coordinates of a ruling of the ruled surface S are defined by six single-valued functions of a parameter s, continuous and possessing five continuous derivatives when s is in the interval Σ. The parameter is assumed to be the projective arc (Hlavaty's π).

Let $p^{ij}(s)$ $(ij = 01, 02, 03, 12, 31, 23)$ and $q^{ij}(s)$ be two sets of six functions meeting the above requirements. The function

$$(1) \qquad p \times q = p^{01}q^{23} + p^{23}q^{01} + p^{02}q^{31} + p^{31}q^{02} + p^{03}q^{12} + p^{12}q^{03}$$

is the cross product of p and q. If $p \times q = 0$ and p is a line, we say p belongs to the complex q. Two complexes p and q for which $p \times q = 0$ are called projectively orthogonal. Under a fundamental correspondence ordering lines and complexes of S^3 to points of S^5, the line p is the image of a point p lying on the absolute quadric V_4^2: $p \times p = 0$. The line or complex and its image point under the fundamental correspondence we call *associated line* (or *complex*) *and point,* and usually denote by the same letter. Projective orthogonality of two complexes is thus equivalent to the conjugacy of their associated points with respect to V_4^2.

As s varies over the interval Σ, the line $p = p^{ij}(s)$ generates the ruled surface S. If primes indicate derivatives, the necessary and sufficient condition that S lie in a constant linear complex is that the matrix $M = \| pp'p'' \cdots \|$ have rank less than 6. S belongs to a constant linear congruence if M is of rank less than 5. We shall assume rank at least 4.

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