HANKEL DETERMINANTS WHOSE ELEMENTS ARE SECTIONS OF A TAYLOR SERIES. PART I.

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1. **Introduction.** This paper contains generalizations of classical theorems on the zeros of sections of a Taylor series, due to Jentzsch [3] and Hurwitz [2]. The methods employed in the proofs are also of independent interest. An application is made of well-known formulas due to Hadamard [1] for the moduli of certain poles of an analytic function. This application appears in §5, and is required for the proofs of Theorem 1 and Theorem 2. The proof of Theorem 3 requires an extension of one of these formulas. The theorem will be stated for completeness, but the proof is of a different nature than the others, and will be presented in Part II.

2. **Statement of the problem.** Let $f(z)$ designate an analytic function which has a branch represented in a neighborhood of $z = \infty$ by a series of negative powers of z,

$$f(z) = \sum_{0}^{\infty} a_k z^{-(k+1)} \qquad (|z| > R, 0 \leq R < \infty). \tag{1}$$

Designate by $s_n(z)$ the n-th section of this power series,

$$s_n(z) = \sum_{0}^{n-1} a_k z^{-(k+1)}. \tag{2}$$

A determinant of order k, with element b_{ij} in the i-th row and j-th column will be denoted by

$$| b_{ij} |_1^k . \tag{3}$$

When the element b_{ij} depends only on $i + j$, this is a Hankel determinant. The problem we consider is that of locating the zeros of the Hankel determinants

$$S_{n,p}(z) = | s_{n+i+j-2}(z) |_1^{p+1} \qquad (n = 1, 2, \cdots), \tag{4}$$

where p is a positive integer or zero. The problem can equally well be stated for series of positive powers of z. The choice of negative powers, however, is desirable at many points in the proofs.

3. **Statement and discussion of results.** For $p = 0$, $S_{n,p}(z)$ reduces to the section $s_n(z)$. This case has been the subject of many investigations. A theorem due to Jentzsch [3] states that every point of the circle of convergence of (1) is a limit point of the zeros of the sections (2). Hurwitz [2] has shown that a

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