ADDENDUM TO "A SIMPLIFIED PROOF OF THE EXPANSION THEOREM FOR SINGULAR SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS"

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The notation used in [1] will be followed here.

In Theorem II of [1], it is shown that to every $f(x)$ of class $L^2(0, \infty)$ there exists a $g(u)$ measurable B such that

$$\int_0^\infty |f(x)|^2 \, dx = \int_{-\infty}^\infty |g(u)|^2 \, d\rho(u). \tag{1}$$

Moreover

$$g(u) = \underset{a\to\infty}{\text{l.i.m.}} \int_0^a f(x)\phi(x, u) \, dx, \tag{2}$$

where convergence in the mean is with respect to $\rho(u)$. Also

$$f(x) = \underset{a\to\infty}{\text{l.i.m.}} \int_{-a}^a g(u)\phi(x, u) \, d\rho(u). \tag{3}$$

In a short paragraph added to [1] before publication, a proof, in the elementary spirit of the paper, was sketched purporting to show that the $g(u)$, which are the transforms of the functions in $L^2(0, \infty)$, fill out the $L^2(\rho)$ space. Professor Ralph Phillips has raised a question about this proof; namely, why is $(-u)^n g(u)$ the transform of $D^n f(x)$, which I cannot now justify. Therefore, the following proof is supplied. Note that the proof uses entirely standard devices but like the proofs of Theorems I and II in [1] is "elementary".

Theorem. *Let $G(u)$ be measurable B and let*

$$\int_{-\infty}^\infty |G(u)|^2 \, d\rho(u) < \infty.$$

Then

$$\underset{a\to\infty}{\text{l.i.m.}} \int_{-a}^a G(u)\phi(x, u) \, d\rho(u) = f(x) \tag{4}$$

exists and moreover if $g(u)$ is the transform of $f(x)$ as given by (2), *then*

$$\int_{-\infty}^\infty |G(u) - g(u)|^2 \, d\rho(u) = 0. \tag{5}$$

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