PROJECTIVE DIFFERENTIAL INVARIANTS OF A CURVE OF A SURFACE

By P. O. Bell

1. **Introduction.** Let S_0 denote a general analytic surface in ordinary projective space and let C denote a curve of S_0 which passes through a general one of its points x_0 . This paper is devoted to the study of (I) the *projective curvature* and the *projective torsion* of C at x_0 , (II) the *projective first fundamental form of* S_0 , and (III) the *projective curvature tensor of* S_0 . A new geometric characterization of each of the above named invariants is presented. In association with the projective curvature tensor and the projective first fundamental form, the projective *total curvature* of S_0 is defined. Upon choosing the R-conjugate line to be the Euclidean normal to S_0 at x_0 and replacing the projective group by its subgroup of orthogonal transformations, the projective first fundamental form, the projective curvature tensor, and the projective total curvature of S_0 actually become the corresponding metric quantities. Moreover, if the Fubini fundamental form is replaced by the first fundamental form in the characterization of the projective curvature of C, the *geodesic curvature of* C *results.* Finally, the covariant points are determined which will be called the *R-centers* of projective principal curvatures of S_0 at x_0 and the *R-center of projective mean curvature* of S_0 at x_0 . Again, *these points become the corresponding metric points when the R-conjugate line is chosen to be the metric normal to* S_0 *at* x_0 .

2. **Differential invariants of a curve of a surface.** Let S be referred to a reference tetrahedron (x_0 , x_1 , x_2 , x_3) whose first three vertices are defined by the vector equations

$$x_0 = x, \qquad x_1 = \frac{\partial x}{\partial u^1}, \qquad x_2 = \frac{\partial x}{\partial u^2},$$

the line l joining the points x_1 , x_2 being covariantly determined with respect to S at x. Let the fourth vertex be a geometrically determined point x_3 . Under an arbitrary transformation of parameters

$$u^\alpha = u^\alpha(\bar{u}^1, \bar{u}^2) \tag{2.1}$$

the vertices are transformed according to the relations

$$\bar{x}_\nu(\bar{u}^1, \bar{u}^2) = x_\nu , \qquad \bar{x}_\beta = x_\alpha \frac{\partial u^\alpha}{\partial \bar{u}^\beta} .$$

Received April 23, 1951.