A NEW APPROACH TO THE STUDY OF CONTACT IN THE PROJECTIVE DIFFERENTIAL GEOMETRY OF SURFACES

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1. **Introduction.** The projective differential geometry of surfaces is studied in the present paper by means of the tensor method introduced by the author in an earlier paper [1; 22–50]. A compact formulation of the projective theory of envelopes [1; 28–29] proves to be an effective tool for the determination of algebraic surfaces having contact of order k with a curve C of an analytic surface S. In this connection, the principal results of the paper are Theorems (3.1) and (3.2). The usual method of finding the equation of an algebraic surface of degree m having contact of order k with a curve C at a point x_0 consists of calculating power series expansions for the equations of C at x_0, of substituting these expansions in an algebraic equation of degree m with undetermined coefficients, and of equating corresponding coefficients of the resulting power series expansions to terms of degree k. The applications of Theorems (3.1) and (3.2) result in great economies of labor, since by these theorems the necessity of performing tedious calculations of power series expansions is eliminated.

2. **Analytic basis.** Consider in ordinary projective space four analytic surfaces S_i, $i = 0, 1, 2, 3$ whose corresponding generic points x_i are linearly independent. The projective homogeneous coordinates of x_i form a square matrix of rank and order four whose elements are analytic functions $x_i^{(p)}$ of two independent variables u^1, u^2. The general coordinates of any point in space may consequently be expressed as a linear combination of the corresponding coordinates of the points x_i. It follows that a set of functions $\Gamma^h_{i\alpha}$ of u^1, u^2, called the connection of the surface S_i, can be uniquely determined such that the functions $x_i^{(p)}$ are solutions of the system of differential equations

$$\frac{\partial x_i}{\partial u^\alpha} - \Gamma^h_{i\alpha} x_h = 0 \qquad (i = 0, 1, 2, 3; \alpha = 1, 2) \tag{2.1}$$

in which h denotes a dummy index. Throughout the present paper, except when a contrary statement is made, Latin indices have the range 0, 1, 2, 3 whereas Greek indices have the range 1, 2; repeated indices in upper and lower positions of adjoining symbols indicate summing over their respective ranges. The most general sets of solutions of (2.1) are the sets of coordinates of the points $\bar{x}_i$ which correspond to the points x_i by a general projective transformation

$$\bar{x}_i^p = c_j^p x_i^j .$$

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