SNAKE-LIKE CONTINUA

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1. **Introduction.** A *chain* is a finite collection of open sets $d_1, d_2, \cdots, d_i$ such that d_i intersects d_j if and only if $i = j - 1, j$, or $j + 1$. We do not impose the condition that the d_i's are connected. If the links of a chain are of diameter less than ϵ, the chain is called an *ϵ-chain.* A compact continuum is called *snake-like* if for each positive number ϵ it can be covered by an ϵ-chain. The term snake-like was suggested to me by Gustav Choquet.

An arc is snake-like while a triod is not. Also, a pseudo-arc [7], [2], [3] is snake-like. If A is the closure of the graph of $y = \sin 1/x$ $(0 < x \leq 1)$, B is the closure of the graph of $y = \sin 1/x$ $(-1 \leq x < 0)$, C is the graph of $y = 1$ $(0 \leq x \leq 1)$, and D is the graph of $y = 0$ $(0 \leq x \leq 1)$, then each of the sets $A, B, C, D, A + B$, and $B + C$ is snake-like. However, $B + D$ is not snake-like because it contains a triod.

A continuum M is called a triod [11] if it contains a subcontinuum N such that $M - N$ is the sum of three mutually separated sets. A continuum which contains no triod is atriodic [8]. Each snake-like continuum is atriodic.

A *circular chain* has more than two links and the first and last links intersect. A finite coherent collection G of open sets is called a *tree chain* if no three of the open sets have a point in common and no subcollection of G is a circular chain. (A collection G is *coherent* if for each proper subcollection G' of it, an element of G' intersects an element of $G - G'$.) A compact continuum is called *tree-like* if for each positive number ϵ there is a tree chain covering it such that each element of the tree chain is of diameter less than ϵ.

A snake-like continuum has a type of linearity that facilitates its study. For example, it has been shown [5] that a snake-like continuum has the fixed point property; that is, if T is a continuous transformation of a snake-like continuum into itself, some point is left fixed. However, it is not known whether or not each bounded plane continuum which does not separate the plane has this property. In fact, it would be of interest to know if each tree-like plane continuum has the fixed point property.

The following three results were proved in [3].

Theorem 1. *If M and N are two nondegenerate snake-like hereditarily indecomposable continua, they are homeomorphic.*

A continuum is indecomposable if it is not the sum of two proper subcontinua. A continuum each of whose subcontinua is indecomposable is called *hereditarily indecomposable.* A continuum homeomorphic with M is called a *pseudo-arc.*

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