A NEW IMPLICATION OF THE YOUNG-POLLARD CONVERGENCE CRITERIA FOR A FOURIER SERIES

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1. **Introduction.** In a series of papers the authors have modified the theory of pointwise Pringsheim convergence of double Fourier series by using the Fréchet variation in place of the classical Vitali variation. The conditions in the classical 2-dimensional tests as listed by Gergen [2] are thereby made less restrictive. A subsequent examination of the implications between the different tests revealed a fundamental gap in the implications between the classical 1-dimensional tests. Specifically it was known previously that the Lebesgue condition L_2 as defined below did not imply the Young-Pollard conditions Y_P ; but it was apparently not known whether the implication $Y_P \Rightarrow L_2$ was valid or not. Pollard had established that $Y_P \Rightarrow L_P$, where L_P denotes the Lebesgue-Pollard conditions, and an example due to Hardy [3; 151] had shown that $L_2 \not\Rightarrow Y_P$.

It is the objective of this paper to establish the implications

$$Y_P \Rightarrow L_2 , \qquad Y' + C_1 \Rightarrow C^0, \tag{1.1}$$

where these conditions are defined below.

We are concerned with an even function f with period 2π, integrable over $[0, \pi]$, and with conditions that the Fourier series for f converge at the origin to a value c^-. The tests are stated in terms of a function φ with values $\varphi(t) = f(t) - c^-$. We refer to three conditions:

C_0
$$\varphi(x) = o(1),$$

C_1
$$\left| \int_0^x \varphi(t)\, dt \right| = o(x),$$

C^0
$$\int_0^x |\varphi(t)|\, dt = o(x),$$

where the principal infinitesimal is $x \geq 0$. We refer to conditions involving the J-variation (J = Jordan) and the L-integral as follows:

Y'
$$\int_0^x |d[t\varphi(t)]| \leq Ax \qquad (0 \leq x \leq \delta)$$

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