SPECIAL CONFORMAL MAPPINGS

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Introduction. The present paper deals with certain special conformal mappings $w = f(z)$ of the unit circle $|z| \leq 1$, in particular with the study of the curves corresponding to the circles $|z| =$ constant and to the radii $\operatorname{arc} z =$ constant.

In Part 1 we compile a set of elementary formulas which are useful in this study.

Part 2 deals with a class of mappings which has been introduced by the first author [4; 61]. In what follows these mappings are referred to as "convex in the vertical direction".

In Part 3 we consider the Cesàro sums of order k of the geometric series:

$$S_n^{(k)}(z) = \binom{n+k}{k} + \binom{n+k-1}{k}z + \binom{n+k-2}{k}z^2 + \cdots + \binom{k}{k}z^n,$$

in particular the sums $S_n^{(1)}(z)$ and $S_n^{(3)}(z)$. The polynomials $S_n^{(3)}(z)$ might be called "tetrahedral polynomials" since the numbers

$$\binom{3}{3}, \binom{4}{3}, \binom{5}{3}, \cdots$$

are often called tetrahedral numbers [2; 4]. The sums $S_n^{(k)}(z)$ occur in the study of the power series whose coefficients are monotonic of a certain order $k + 1$. The mapping $w = S_n^{(k)}(z)$ has been considered for various values of k. [See list of References.] In Part 3 some of these results are proved in a new way and refined by new results.

PART 1.

1. For the sake of completeness we point out a set of simple formulas; at least part of them will be used in the later discussion.

Let $z = x + iy = re^{i\theta}$, $w = f(z) = u(r, \theta) + iv(r, \theta)$. We denote the derivatives of u and v with respect to θ by u', v', and with respect to r by $\dot{u}$, $\dot{v}$, respectively. We have then:

$$u + iv = f(z), \tag{1}$$

$$u' + iv' = izf'(z), \tag{2}$$

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