CONJUGATE NETS IN THREE- AND FOUR-DIMENSIONAL SPACES

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Introduction. The purpose of this paper is to make some contributions to the projective differential geometry of conjugate nets in three- and four-dimensional spaces.

§1 contains a summary of the analytic basis for the development of the first chapter devoted to the study of a conjugate net in ordinary space. Let Φ be a fixed plane and P_x be a nonsingular point of a conjugate net N_x in ordinary space. The points M, M' of intersection of the fixed plane Φ and the two tangents of the net N_x at the point P_x describe two plane nets N_M, $N_{M'}$ respectively. In §2, we show that one of the two plane nets N_M, $N_{M'}$ is a Laplace transformed net of the other, and we also study a special case in which one of the two plane nets N_M, $N_{M'}$ has equal and nonzero Laplace-Darboux invariants.

The second chapter treats of a conjugate net N_x in a four-dimensional space S_4. §3 contains a summary of the analytic basis for the development of this chapter. In §4 some of the results obtained in §2 are extended to the space S_4 by using a fixed hyperplane instead of the fixed plane Φ. Let Ψ be a fixed plane determined by two fixed hyperplanes in the space S_4, and N_T be the plane net described by the point T of intersection of the fixed plane Ψ with the tangent plane at a point x of the net N_x. In the last section, we derive the equation of Laplace and the Laplace-Darboux invariants for the plane net N_T, and also study some special cases in which one or both of the first and minus-first Laplace transformed nets of the net N_T degenerate into curves or the net N_T has equal and nonzero Laplace-Darboux invariants.

I. Conjugate Nets in Ordinary Space

1. **Analytic basis.** Let N_x be a conjugate net with parameters u, v on an analytic proper surface S in ordinary space. For the sake of convenience we take the conjugate net N_x on the surface S as parametric, so that the homogeneous projective coordinates $x^{(1)}, \cdots, x^{(4)}$ of a point P_x on the surface S are given as analytic functions of the two independent variables u, v by equations of the form

$$x = x(u, v). \tag{1.1}$$

The four coordinates x and the four coordinates y of the point P_y, which is the harmonic conjugate of the point P_x with respect to the foci of the axis of

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