SOME RESULTS CONCERNING HOMOGENEOUS PLANE CONTINUA

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1. **Introduction.** A set M is called *homogeneous* if, for each pair of its points x, y, there exists a homeomorphism of M into itself which carries x into y. Shortly after this definition was introduced by Sierpinski in 1920 [9], Knaster and Kuratowski proposed this problem: Does there exist a homogeneous, nondegenerate, bounded plane continuum other than a simple closed curve? [4].

A partial solution was given in 1924 by Mazurkiewicz, who showed that the simple closed curve was the only homogeneous, bounded plane continuum which was also locally connected [5]. Some years later, Waraskiewicz claimed that the answer to the question of Knaster and Kuratowski was in the negative [10]; several consequences of this statement were subsequently announced by Choquet [2].

However, in 1948, Bing exhibited a homogeneous, bounded plane continuum which was *not* a simple closed curve [1]. (This continuum had previously been used by E. E. Moise as the first example of a bounded plane continuum which was homeomorphic to each of its nondegenerate subcontinua, and which was not an arc [6].) In disposing of the above question, this result immediately raised new ones, some of which form the basis for this paper.

In particular, we shall see that the result of Mazurkiewicz can be strengthened in two alternative directions. First, it will be shown that a locally connected, bounded plane continuum must be a simple closed curve even if it is *locally* homogeneous. On the other hand, instead of weakening the hypothesis at this point, we may replace the local connectedness of M by the condition that M merely contain a simple closed curve. It will also follow, from this second result, that a homogeneous, bounded plane continuum which is arcwise connected must be a simple closed curve.

In the final section, it will be shown that any bounded plane continuum consisting of the sum of a collection of disjoint simple closed curves must be topologically equivalent to a closed annulus. We shall see that this fact has a close connection with the second strengthening of the theorem of Mazurkiewicz.

This paper contains the substance of a thesis written under the direction of R. H. Bing, and much of it is the result of his suggestions and criticism. The author wishes to express his appreciation for the continued encouragement and interest shown by Professor Bing.

2. **Definitions.** In general, the usage followed is that of R. L. Moore [7]. Some terms which have been employed by other authors with somewhat different meanings are included below.

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