SUMS OF PRODUCTS OF POLYNOMIALS IN A GALOIS FIELD

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1. **Introduction.** In [2] the problem of finding the number of representations of a polynomial F in the ring $GF[p^n, x]$ as a sum of products

$$F = \alpha_1 X_1^s Y_1 + \cdots + \alpha_\nu X_\nu^s Y_\nu \qquad (\alpha_i \in GF(p^n)) \tag{1.1}$$

was considered. In this paper (§4) we show how certain types of (non-trivial) polynomial coefficients can be introduced in (1.1) with results similar to those of [2]. In §3, as a preliminary to this problem, we generalize the original result for constant coefficients using the fundamental theorem on arithmetic functions due to Carlitz [1]. This theorem we restate as follows: *Any* function $f(A)$ can be expressed *uniquely* in the form

$$f(A) = \sum{}^{*} a_{GH}\epsilon_{GH}(A), \qquad a_{GH} = p^{-nr} \sum f(B)\epsilon_{GH}(-B), \tag{1.2}$$

where the sum $\sum^*$ is over a fundamental set of ϵ's, [1, §3] and where the interior sum is over B of degree $< r$ if deg $A < r$ and over B of degree $= r$, sgn $B = \alpha$, if deg $A = r$, sgn $A = \alpha$. In §2 certain sums required in the succeeding sections are evaluated. These sums are of some interest within themselves and are evidently of importance in connection with several problems mentioned in §§3 and 4.

2. **Evaluation of sums.** The function $f(A) = \delta_z^s(A)$ is defined as in [2; §4] to be the number of primary D of degree z such that $D^s \mid A$. We now show how this function for certain z can be expressed in terms of the ϵ's.

First we suppose deg $A < r = 2ks + t$ ($t \geq 0$, $k \geq z$). Using (1.2) we get

$$f(A) = p^{-nr} \sum{}^{*} \epsilon_{GH}(A) \sum_{\substack{\deg U = z,\ \operatorname{sgn} U = 1 \\ \deg V < r - zs}} \epsilon_{GH}(-U^s V) = \sum\nolimits_1 + \sum\nolimits_2 , \tag{2.1}$$

where

$$\sum\nolimits_1 = p^{-nr} \sum_{\substack{(G,H)=1 \\ h \leq zs}} \epsilon_{GH}(A) \sum \epsilon_{GH}(-U^s V) \tag{2.2}$$

and

$$\sum\nolimits_2 = p^{-nr} \sum_{zs < h \leq r}{}^{*} \epsilon_{GH}(A) \sum \epsilon_{GH}(-U^s V), \tag{2.3}$$

the sum in $\sum_1$ being over all $\epsilon_{GH}(h \leq zs)$ since no two ϵ's of this set are equivalent.

We note by [1; (3.2)] that $\sum_1 = 0$ except for those values of H and U for which $H \mid U^s$. In other words, H must be of the form $H = L^s M$, where M has

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