DECOMPOSITIONS INDUCED UNDER FINITE-TO-ONE CLOSED MAPPINGS

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1. **Introduction.** Let X be a separable metric space of dimension n. A *decomposition* of the space X is a finite collection α of closed sets F_i whose sum is X. α is called an ϵ-decomposition if no F_i is of diameter greater than ϵ. A decomposition α is said to be *regular* if the intersection of each j of the sets F_i is of dimension at most $n - j + 1$. A set of the form $F_{i_1} \cdots F_{i_t}$, where the F_{i_j} $(j = 1, \cdots, t)$ are distinct sets of the collection α, is called a *t-fold intersection* of α. Such a set is closed and, if α is regular, is of dimension at most $n - t + 1$. The subset of a t-fold intersection consisting of points which belong to the t-sets defining the t-fold intersection and to no other sets of α is called a *t-set* of α. A t-set of α is an F_σ and, if α is regular, is of dimension at most $n - t + 1$. No point of X may belong to more than one t-set.

A *mapping* is a continuous transformation. A mapping f of a space X onto a space Y is said to be *closed* if for each closed subset C of X we have $f(C)$ closed in Y. Any mapping of a compact space is a closed mapping. Under a closed mapping $f(X) = Y$ the closed sets F_i of a decomposition α of X will go into closed sets F'_i which form a decomposition α' of Y. We shall say that α' is the decomposition of Y induced by the decomposition α under the mapping f. L. V. Keldys has proved the following lemma [4]. *Let X be a compactum* (compact metric space), $f(X) = Y$, *f continuous. If there exists an ϵ-decomposition of Y of order m then there exists a regular decomposition of X which induces in Y an ϵ-decomposition of order m.* This lemma is stated in more general form in §6 and a less complicated proof is given. The theorem proved in this paper is of a similar nature.

2. **Statement of the theorem.** It is known that if a separable metric space X is contained in the sum of a finite number of open sets U_i there exists a regular decomposition of X into closed sets each of which is contained in at least one of the sets U_i [5; 161]. The main theorem is concerned with finite-to-one closed mappings of such a space X onto a space Y which may be thought of as lying in some Euclidean space. f is said to be a *k-to-one* mapping if, for each $y \in Y$, $f^{-1}(y)$ contains at most k points of X. We now state the theorem.

Theorem. *Let X be a separable metric space of dimension n. Let $X = U_1 + \cdots + U_m$, U_i open in X $(i = 1, \cdots, m)$, m finite. Let $f(X) = Y$, where f is a k-to-one closed mapping. Then there exists a regular decomposition α of X into*

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