CONVERGENCE AND (C, 1, 1) SUMMABILITY OF DOUBLE ORTHOGONAL SERIES

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1. Introduction. Let E be the Cartesian product of two sets E_1 and E_2, where E_k $(k = 1, 2)$ has finite measure and is embedded in Euclidean space. If $\{\phi_m\}$ $(m = 1, 2, \cdots)$ is a complete orthonormal system (CONS) of functions of class L^2 on E_1, and $\{\psi_m\}$ a similar set on E_2, then $\{\phi_{mn} = \phi_m\psi_n\}$ $(m, n = 1, 2, \cdots)$ forms a CONS on E; that is,

$$\int_E \phi_{mn}\phi_{pq}\, dA = \delta_{mp}\delta_{nq} \qquad \text{(ON relations)}, \tag{1.1}$$

where dA is the Euclidean volume element on E, and $\int_E \phi_{mn}f\, dA = 0$ $(m, n = 1, 2, \cdots)$ for an $f \in L^2$ implies $f = 0$ almost everywhere (a. e.) on E (completeness).

The orthogonal development of any function $f \in L^2$ with respect to the ONS $\{\phi_{mn}\}$ is given by

$$\sum_{m,n=1}^{\infty} a_{mn}\phi_{mn}, \tag{1.2}$$

where

$$a_{mn} = \int_E \phi_{mn} f\, dA. \tag{1.3}$$

The mn-th partial sum of series (1.2) will be denoted by S_{mn} and the mn-th $(C, 1, 1)$ sum by σ_{mn}. The mn-th kernel function is

$$K_{mn}(P, Q) = \sum_{j,k=1}^{m,n} \phi_{jk}(P)\phi_{jk}(Q) \qquad (P, Q \in E), \tag{1.4}$$

and the corresponding Lebesgue function is

$$L_{mn}(P) = \int_E |\, K_{mn}(P, Q)\,|\, dA. \tag{1.5}$$

For $(C, 1, 1)$ summability the analogous functions are

$$\begin{aligned} K_{mn}^{(1)}(P, Q) &= \frac{1}{mn}\sum_{j,k=1}^{m,n} K_{jk}(P, Q) \\ &= \sum_{j,k=1}^{m,n}\left(1 - \frac{j-1}{m}\right)\left(1 - \frac{k-1}{n}\right)\phi_{jk}(P)\phi_{jk}(Q), \end{aligned} \tag{1.6}$$

$$L_{mn}^{(1)}(P) = \int_E |\, K_{mn}^{(1)}(P, Q)\,|\, dA. \tag{1.7}$$

Similar notation is used in referring to the simple orthogonal series.

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