MAPPING BY p-REGULAR FUNCTIONS

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1. **Principal Theorem.** Let a be a positive number. Let Γ denote the circle of radius a about the origin as center in the (x,y)-plane. Let S be the interior of Γ; and let $\bar{S} = \Gamma + S$. We consider a function $p(x, y)$ satisfying the following conditions on S and $\bar{S}$.

$$(1.1) \qquad p \in C^0(\bar{S}); \qquad p \text{ is real and } > 0 \text{ on } \bar{S}.$$

$$(1.2) \qquad p \in C'(S); \qquad p_x, \; p_y \text{ are bounded on } S.$$

By the notation $p \in C^0(\bar{S})$ is meant that $p \in C^0$ on $\bar{S}$. A function $F(z) = u(x, y) + iv(x, y)$, $z = x + iy$, u and v real, is said to be *p-regular* in S if $F \in C'(S)$; *i.e.*, if $u, v \in C'(S)$, and if on S

$$(1.3) \qquad pu_x = v_y, \qquad pu_y = -v_x.$$

The *p-derivative*, F', of F is defined as

$$F' = p^{\frac{1}{2}}u_x + iv_x p^{-\frac{1}{2}}.$$

This derivative satisfies

$$|F'|^2 = p\,|\nabla u|^2 = |\nabla v|^2/p = \partial(u, v)/\partial(x, y).$$

A function u which is the real part of a p-regular function is *p-harmonic*. Evidently, if $u \in C''(S)$, then u is p-harmonic in S if, and only if, on S

$$(pu_x)_x + (pu_y)_y = 0.$$

Any complex [real] constant is seen to be p-regular [p-harmonic]. Any real linear combination of p-regular [p-harmonic] functions is p-regular [p-harmonic]. The product of i and a p-regular function is $(1/p)$-regular. The imaginary part of a p-regular function is $(1/p)$-harmonic. For the case $p \equiv 1$, a p-regular [p-harmonic] function reduces to an analytic [harmonic] function and the p-derivative to the ordinary derivative.

Our chief purpose in this paper is to prove the following generalization of the Riemann mapping theorem.

THEOREM 1. *Let S' be a finite domain in the Z-plane whose boundary Γ' is a simple closed rectifiable curve. Let $z^{(1)}$, $z^{(2)}$, $z^{(3)}$ be distinct points on Γ; and let $Z^{(1)}$, $Z^{(2)}$, $Z^{(3)}$ be distinct points on Γ' in the same order on Γ' as the points $z^{(1)}$,*

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