A THEOREM OF FÉDOROFF

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1. **Introduction.** Let $f(z)$ be a continuous function for $z = x + iy$ in the unit disc $\mathfrak{D}$: $|z| < 1$, and let $D(z, r)$ denote the closed circular disc with center z, radius r, and boundary $C(z, r)$. Then a result due to Fédoroff [3; 512] states that a necessary and sufficient condition that $f(z)$ be analytic in $\mathfrak{D}$ is that the equation

$$\iint_{D(z,r)} (\zeta - z) f(\zeta)\, d\xi\, d\eta = 0 \tag{1}$$

hold for each $D(z, r)$ in $\mathfrak{D}$. This is analogous to a form of the Cauchy and Morera theorems that states that a necessary and sufficient condition that $f(z)$ be analytic in D is that the equation

$$\int_{C(z,r)} f(\zeta)\, d\zeta = 0 \tag{2}$$

hold for each $C(z, r)$ in $\mathfrak{D}$.

In this note we obtain analogues of the preceding results for areolar monogenic functions; these results are along the lines suggested by a recent note due to Haskell [4]. Then we investigate briefly the implications of equations (1) and (2) when the circular domains are replaced by certain polygonal domains; these results generalize earlier results due to the present author [9].

Although areolar monogenic functions go back to Pompéiu [7], we shall cite only the more recent literature. Ridder [10] and Kriszten [5] have excellent bibliographies.

2. **Circular domains.** If $f(z)$ has continuous partial derivatives of the first order in $\mathfrak{D}$, then a necessary and sufficient condition that $f(z)$ be analytic in $\mathfrak{D}$ is that the Cauchy-Riemann equations hold in $\mathfrak{D}$:

$$\lambda f(z) \equiv \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) f(z) \equiv \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0. \tag{3}$$

We say that $f(z)$ is *areolar monogenic* in $\mathfrak{D}$ if and only if $\lambda f(z)$ is an analytic function in $\mathfrak{D}$. Hence it follows that $f(z)$ is areolar monogenic if and only if the condition

$$\lambda(\lambda f(z)) \equiv \lambda^2 f(z) = 0 \tag{4}$$

holds in $\mathfrak{D}$.

It follows at once that if $f(z)$ is areolar monogenic in $\mathfrak{D}$, then $f(z)$ has partial derivatives of all orders in $\mathfrak{D}$.

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