THE STRUCTURE OF LOCALLY COMPACT GROUPS

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Since the introduction of Haar measure great strides have been made toward understanding the structure of locally compact groups. Perhaps the most striking fact yet discovered is the close relationship which exists between Lie groups and certain special classes of locally compact groups; *viz.*, compact, Abelian or solvable groups. It is widely conjectured that similar relationships hold in general. In this paper we shall prove several theorems which strengthen this conjecture and reduce its verification to the study of simple groups.

§§1 and 2 contain preliminaries. Our first main result, given in §3, is that a group with a Lie normal subgroup and corresponding Lie quotient group is itself a Lie group. In §4 we define and study a class of groups which we call generalized Lie groups; here the most important is the extension theorem similar to that obtained for Lie groups. In §5 we introduce the concept of the height of a group, which is in some respects analogous to the dimension but which is finite for all locally compact groups. §6 contains a proof that every connected locally compact group has a largest solvable normal subgroup and that the corresponding quotient group is a direct product of indecomposable groups. §7 is devoted to a brief discussion of the relation between the results and the conjecture suggested above.

References have been given only to a few relatively recent papers. Where no other reference is given, the reader is referred to Weil's excellent book [8] which contains almost all the results required but not proved in this paper.

Theorem 3.1 and consequently much of the rest of the paper depend on a theorem recently proved by Kuranishi [5]; the author was privileged to see Kuranishi's paper prior to its publication. After the author had proved the theorems of §§3, 4, and 6 (except 6.17) it was brought to his attention that Iwasawa has recently obtained many similar results [3].

1. Conventions and preliminary results.

1.1. In general our notations will agree with those of Weil [8]. In particular, we call attention to the following conventions: By a neighborhood of a point p we mean any set of which p is an interior point. We do not require that a subgroup of a topological group be closed. A continuous mapping of one topological group into another is called a representation if it is an algebraic homomorphism, the term homomorphism being reserved for the case that the mapping is open as well as continuous.

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