THE GIBBS PHENOMENON AND BOCHNER'S SUMMATION METHOD (II)

By Min-Teh Cheng

1. In a previous note [3] we have investigated the Gibbs set of the Riesz mean of the Fourier series of a class of functions in one variable. It turns out that the method used there can be extended to a class of functions of several variables. We shall deal with the Gibbs phenomenon for double Fourier series for the sake of simplicity. The essential fact used in our proof will be the monotonic property of Bessel functions (Lemma 1 below). A rigorous proof of this lemma was first given by Cooke [4], [5]; it depends on a calculation of the zeros of Bessel functions and is rather long. It should be mentioned here that in our previous note we have avoided using this fact by establishing directly Lemmas 4 and 5 in a much simpler manner. It should also be noticed that Lemma 4 of our previous note still suffices to render our present results for the two-dimensional case. However, our method can be extended to functions of more than two variables, and for this extension Cooke's result would be needed, anyway.

We shall first of all define the Gibbs phenomenon for a sequence of functions of two variables in the following manner.

Let $\{f_n(x, y)\}$ be a sequence of real-valued functions defined in a neighborhood of a point (ξ, η) in the Euclidean plane. Suppose that $\{f_n(x, y)\}$ converges to a function $f(x, y)$ for $\xi < x \leq \xi + h$, $\eta < y \leq \eta + k$ and that $f(\xi + 0, \eta + 0)$ exists, meaning that, when (x, y) tends to (ξ, η) from within the 'first quadrant' $(x > \xi, y > \eta)$, the limit of $f(x, y)$ exists. Then we say that $\{f_n(x, y)\}$ presents Gibbs phenomenon in the RR- neighborhood of the point (ξ, η), whenever

$$\limsup_{\substack{(x,y)\to(\xi+0,\eta+0)\\ n\to\infty}} f_n(x, y) > f(\xi + 0, \eta + 0),$$

$$\liminf_{\substack{(x,y)\to(\xi+0,\eta+0)\\ n\to\infty}} f_n(x, y) < f(\xi + 0, \eta + 0).$$

We say that $\{f_n(x, y)\}$ presents Gibbs phenomenon in the RL-neighborhood of the point (ξ, η), if $\{f_n(x, y)\}$ converges to $f(x, y)$ for $\xi < x \leq \xi + h$, $\eta - k \leq y < \eta$ and

$$\limsup_{\substack{(x,y)\to(\xi+0,\eta-0)\\ n\to\infty}} f_n(x, y) > f(\xi + 0, \eta - 0),$$

$$\liminf_{\substack{(x,y)\to(\xi+0,\eta-0)\\ n\to\infty}} f_n(x, y) < f(\xi + 0, \eta - 0),$$

where $f(\xi + 0, \eta - 0)$ is supposed to be existent.

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