EXTENSIONS OF THE PONTRJAGIN DUALITY
II: DIRECT AND INVERSE SEQUENCES

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1. Introduction. In a previous paper on this subject [1] we showed that the class of groups satisfying the Pontrjagin Duality is closed under the operation of taking infinite products. Another question in the same general problem is whether the class is closed under the operation of taking inverse limits. It has been known for a long while that an inverse limit of compact groups does satisfy the Pontrjagin Duality. In that case the character group is a direct limit of discrete groups [2; Chapter II, (20.8)]. Consequently, it is to be expected that the solution of the general question will involve first giving a suitable definition of direct limit for groups which need not be discrete. We do this in §3. Using the definition given there, we show in the present paper that the limit group of a countable inverse sequence of locally compact groups satisfies the Pontrjagin Duality and that its character group is the limit of a direct sequence. Specifically, we prove the following theorem.

Let $\{G_n\}$ $(n = 1, 2, \cdots)$ *be an inverse sequence of locally compact groups and* $\{H_n\}$ *the "dual" direct sequence of the corresponding character groups. Then the inverse limit of the* G_n's *and the direct limit of the* H_n's *are character groups of each other.*

The question of whether such a duality theorem holds for general direct and inverse systems (even of locally compact groups) remains still unanswered. In §8 we discuss some of the problems involved in attacking this question.

The outline of the paper is briefly as follows. In §2 we show that the (full) infinite products of locally compact groups retain two important properties of such groups, *viz.* the existence of sufficiently many characters and the extendability of characters of subgroups. In §3 we define the limit of a direct system of groups and show that, as far as studying the limit group is concerned, the given system can be replaced by a relatively simple one. In §4 we discuss dual inverse and direct systems and prove some preliminary properties of such systems. In §5 we prove the above duality theorem, and in §§6 and 7, we establish two basic lemmas used in the proof of the duality theorem. §8 is devoted to some final comments on the paper and, as we mentioned, on the problems involved in a more general duality theorem.

During the period of working on the present paper, the author had the benefit of repeated discussions with Professor N. E. Steenrod. The results obtained

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