CONCERNING ABSTRACT SPACES

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Introduction. Moore has shown [3] that if Axioms 0 and 1 stated below hold true and there do not exist uncountably many mutually exclusive domains, then space is separable. However, if Axiom 1_3 denotes Axiom 1 with part (4) deleted, Theorem 1 results. Part I of this paper is concerned with this result.

Part II is concerned with the relations between several axioms similar to Axiom 1. In particular, three modifications of Axiom 1 are denoted as Axioms $1'$, $1''$, and $1'''$, and it is shown that, in the presence of Axiom 0, Axioms 1 and $1'$ are equivalent, Axioms $1''$ and $1'''$ are equivalent, and Axiom 1 implies Axiom $1''$ but not conversely.

I. Axioms 0 and 1 of [4] are as follows.

Axiom 0. *Every region is a point set.*

Axiom 1. *There exists a sequence $G_1, G_2, G_3, \cdots$ such that* (1) *for each n, G_n is a collection of regions covering S,* (2) *for each n, G_{n+1} is a subcollection of G_n,* (3) *if R is any region whatsoever, X is a point of R and Y is a point of R either identical with X or not, then there exists a natural number m such that if g is any region belonging to the collection G_m and containing X then $\bar{g}$ is a subset of $(R - Y) + X$,* (4) *if $M_1, M_2, M_3, \cdots$ is a sequence of closed point sets such that, for each n, M_n contains M_{n+1} and, for each n, there exists a region g_n of the collection G_n such that M_n is a subset of $\bar{g}_n$ then there is at least one point common to all the point sets of the sequence $M_1, M_2, M_3, \cdots$.*

Theorem 1. *There is a space satisfying Axioms* 0 *and* 1_3 *which is not separable and does not contain uncountably many mutually exclusive domains.*

Proof. For each positive integer x, let I_x denote the sequence of points $(x, 1)$, $(x, \frac{1}{2})$, $(x, \frac{1}{3})$, $\cdots$. There is an uncountable well ordered sequence b of infinite sequences of points such that: (1) no term of b is preceded by uncountably many others, (2) if $Y_1, Y_2, Y_3, \cdots$ is a sequence Y of b, then Y_i belongs to I_i, and (3) if $Z_1, Z_2, Z_3, \cdots$ follows Y in b, there is an n such that, for i greater than n, Z_i is below Y_i.

There is a well ordered sequence c of rays such that (1) the ray Y belongs to c if and only if there is a sequence $Y_1, Y_2, Y_3, \cdots$ of the sequence b such that Y is made up of the points $Y_1, Y_2, Y_3, \cdots$ and the points of the segments Y_1Y_2; Y_2Y_3; Y_3Y_4; $\cdots$, and (2) if X precedes Z in b, then the ray of c which contains all the points of X precedes the one which contains all the points of Z.

A sensed pair $(e; Y)$ is a *breakdown* if e is a finite sequence $e_1, e_2, \cdots, e_i$ of

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