HOMOTHETIC CORRESPONDENCES BETWEEN RIEMANNIAN SPACES

By E. Baylis Shanks

1. **Introduction.** In the literature concerning the intrinsic theory of Riemannian spaces, most writers refer to homothetic correspondences only as special cases of more general correspondences, in particular, conformal and geodesic correspondences. On the other hand, the theory of isometric correspondences (which are a proper subset of the class of homothetic correspondences) has received considerable attention.

It is the purpose of this paper to investigate the theory of homothetic correspondences between Riemannian spaces, in which the theory of isometric correspondences is contained as a special case.

2. **Definition and characterization of homothetic correspondences.** Let V_n be an n-dimensional Riemannian space, whose metric is defined by the real fundamental quadratic form

$$ds^2 = g_{ij}\, dx^i\, dx^j \tag{1}$$

where the summation convention (for notation and convention, see [5]) of tensor calculus is used and $i, j = 1, \cdots, n$. (Unless otherwise stated, indices will take values $1, \cdots, n$.) We assume the form (1) is positive definite and g_{ij} (for all i and j) is a real and continuous function of real coordinates $x^1, \cdots, x^n$, as well as its derivatives of all orders appearing in the discussion, in a domain in which the implicit equations (2) below have unique solutions for either set of coordinates as a function of the other set.

We define a homothetic correspondence between V_n and V_n^* with the linear element $g_{ij}^*\, dx^{*i}\, dx^{*j}$ as a 1-1 point correspondence given by n independent equations of the form

$$F_i(x) = F_i^*(x^*), \tag{2}$$

where $F_i(x)$ is an abbreviated notation for $F_i(x^1, \cdots, x^n)$, such that corresponding distances on V_n and V_n^* are always in the same constant ratio.

If, then, we interpret equations (2) as a change of coordinates on V_n and denote the fundamental tensor of V_n after the change of coordinates by $g'_{ij}(x^*)$, we will have the identity

$$g_{ij}^* = e^{2a} g'_{ij}\,, \qquad (a = \text{constant}). \tag{3}$$

Received September 29, 1948. This paper is an abstract of a thesis prepared under the direction of Professor H. Levy (to whom the author wishes to express appreciation) at the University of Illinois in 1947. Presented to the American Mathematical Society, December 30, 1947.