THE ASYMPTOTIC BEHAVIOR OF THE MINIMUM IN A SEQUENCE OF RANDOM VARIABLES

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1. **Introduction.** Let $\xi_n = \min_{k \leq n} X_k$, $n = 1, 2, \cdots$, where $X_1, X_2, \cdots$ is a sequence of mutually independent random variables whose respective distribution functions, assumed to be non-unitary throughout this work, are $F_1(x), F_2(x), \cdots$.

If sequences of real constants (norming parameters) $\{a_n > 0\}$ and $\{b_n\}$, $n = 1, 2, \cdots$, exist such that

$$\lim_{n \to \infty} \Pr \{\xi_n > a_n x + b_n\} = \lim_{n \to \infty} \prod_{k=1}^{n} [1 - F_k(a_n x + b_n)] \tag{1}$$

exists and is of the form $1 - V(x)$ where $V(x)$ is a non-unitary distribution function, then one says that $\{\xi_n\}$ has a limiting distribution. It is known that the limit class $V(ax + b)$, if it exists, is unique. Furthermore, for each non-unitary limit $V(x)$, the sequences $\{a_n > 0\}$ and $\{b_n\}$ are asymptotically unique (see [3; Lemma 1]). We shall give some conditions under which $\{\xi_n\}$ has a limiting distribution.

Completing previous work by Fréchet [2], Fisher and Tippett [1], and de Misès [5] in the case of identically distributed variables $\{X_k\}$, Gnedenko [3] has shown that there exist only three classes of limiting distributions and has given necessary and sufficient conditions for convergence to each class. One of the chief purposes of this work is to point out, in §3, that much more general classes exist if we drop the restriction that the random variables $\{X_k\}$ are identically distributed.

In our case if no restrictions on $\{F_k(x)\}$ are imposed, then it may happen that the behavior of (1) is determined by a finite number of random variables of the sequence $\{X_k\}$. We would then not have a limit theorem in the usual sense of the word. As an example, consider the case where $F_k(x) = V(k^k x)$, $k = 1, 2, \cdots$, where $V(x)$ is any distribution function such that $V(0) = 0$, $V(x) > 0$ for $x > 0$, and, as $x \to 0$, $V(x) = o(x^\epsilon)$, where $\epsilon > 0$. It is clearly necessary that $b_n \sim 0$ and $a_n = O(n^{-n})$ in order for (1) to exist and be non-unitary. In this case the behavior of (1) is completely determined by the contribution from the single term $F_n(a_n x + b_n)$. Similarly the behavior of (1) is essentially determined by a finite number of elements of the sequence $\{X_k\}$ whenever $\sum F_k(x) < \infty$. In order to eliminate such cases we shall use conditions

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