NOMOGRAPHIC DISJUNCTION

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1. **Introduction.** Necessary and sufficient conditions that a given function $F(x, y, z)$ equal a determinant of the form

$$(1.1)\qquad \begin{vmatrix} p_1(x) & q_1(x) & r_1(x) \\ p_2(y) & q_2(y) & r_2(y) \\ p_3(z) & q_3(z) & r_3(z) \end{vmatrix}$$

and processes for finding such a determinant when one exists have been stated by E. Duporcq, by O. D. Kellogg and by D. F. Barrow. In contrast to the others, Duporcq's treatment does not use derivatives, but it is incomplete (see §6).

The present paper aims, in the first place, to give a discussion which covers all cases and which maintains the advantages of Duporcq's view-point, namely, thoroughgoing exploitation of the idea of linear dependence and avoidance of derivatives. In the second place, the present paper aims to present the theory so that it is readily applicable. With this in mind, our general plan is to find with the least possible calculation a determinant which must be a representation of F, if there is one. To complete the work, it is sufficient to test whether F is identically equal to that determinant. Thus only one identity has to be tested instead of Barrow's nine, for example, and that identity is one whose verification is required by practical considerations in any case. Accordingly, we attach little importance to the circumstance that necessary and sufficient conditions can be inferred from our discussion.

Eliminating the derivatives from the discussion is advantageous because it removes the necessity of calculating them. The extension of the function domain is at best of secondary importance. If the derivatives exist and are easily calculated, they can be employed in the disjunction.

2. **Function rank.** The constants employed are from a commutative field, which in applications usually consists of the real numbers. The *variable domain* consists of all triples (x, y, z) which arise as each of the variables x, y, z varies independently over a corresponding given set of values. Each function has for each (x, y, z) in the variable domain a single value from the field of constants. Two functions are (identically) *equal* if and only if they are equal for every (x, y, z) in the variable domain.

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