AN EXTREMAL PROPERTY OF MONOTONE-LIGHT FACTORIZATIONS

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1. Introduction.

1.1. In this paper the topological structure theory of Peano spaces is applied to the isoperimetric inequality. For concepts and results relating to this theory we use as a general reference the book of G. T. Whyburn on Analytic Topology [4] and the book of T. Radó on Length and Area [3]. This latter book will be referred to as [LA].

1.2. Let U denote the positively oriented unit sphere $u^2 + v^2 + w^2 = 1$ in a Euclidean uvw-space and let T be a continuous mapping from U into a Euclidean xyz-space E_3. Then T may be thought of as a representation of an F-surface of the type of the 2-sphere (see [LA; II.3.44]). Let us denote by $A(T)$ the Lebesgue area of this surface (see [LA; V.2.3]). Then $A(T)$ may be thought of as a functional of the continuous mapping T.

1.3. Radó [2] has introduced the following concepts and results relating to the isoperimetric inequality. For a continuous mapping T (see 1.2) let $i(x, y, z; T)$ be the topological index-function associated with the mapping T (see Alexandroff-Hopf [1]). The volume $V(T)$ enclosed by the surface represented by T is defined to be

$$(1) \qquad V(T) = \begin{cases} \iiint \mid i(x, y, z; T) \mid dx\, dy\, dz & \text{if } i(x, y, z; T) \text{ is summable,} \\ +\infty & \text{otherwise,} \end{cases}$$

where the triple integral is extended over the whole xyz-space E_3. Then $V(T)$ may be thought of as a functional of the continuous mapping T. For the Lebesgue area $A(T)$ and the enclosed volume $V(T)$ we have then (see Radó [2]) the isoperimetric inequality

$$(2) \qquad V(T)^2 \leq A(T)^3/36\pi.$$

1.4. In a forthcoming paper on the isoperimetric inequality J. W. T. Youngs considers *unrestricted factorizations* of a mapping T in the following sense. T is the product of two continuous mappings f and s, where f is a continuous mapping from U into a Peano space $\mathfrak{M}$ and s is a continuous mapping from $\mathfrak{M}$ into E_3. For such an unrestricted factorization let C be a generic notation for a proper cyclic element of $\mathfrak{M}$ (see [LA; II.2.10], [LA; II.2.50]) and let r_C

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