ENDELEMENTS AND THE INVERSION OF CONTINUA

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In this note X and Y will always denote compact (= bicompact) connected Hausdorff spaces. We designate by f a map (= single-valued continuous transformation) of X onto Y.

It will be shown that, if f is non-alternating, there exists a proper subcontinuum of Y with a connected inverse. This continuum may be a point of Y. To accomplish this end we extend two theorems due to G. T. Whyburn [4; 77, (8.1)] and [3].

We recall briefly some definitions and notations (Wallace [1]). If the sets A and B are separated we write $A \mid B$. For any pair of points p and q, not separated in the space in question by a single point, we write $p \sim q$. A prime-chain is a continuum which is either an endpoint, a cutpoint or a nondegenerate set E containing a and b, with $a \sim b$, and representable as $E = \{x \mid a \sim x \sim b\}$. In a Peano space the prime-chains are exactly the cyclic elements (Whyburn [4]). By an *endelement* is meant a prime-chain E with the property that, if U is open and contains E, then there is an open set V with $E \subset \overline{V} \subset U$ and $F(V) = \overline{V} - V$ = a single point. In a Peano space such sets are either endpoints or nodes.

Theorem 1. *If X contains a cutpoint it contains an endelement.*

Proof. Let K be the set of all cutpoints of X and $q \in X - K$. For $k, k' \in K$ we define $k < k'$ to mean that k' separates k and q in X. We then have:

(1) The relations $k < k'$ and $k' < k$ are inconsistent.

(2) $k < k'$ and $k' < k''$ imply $k < k''$.

Let M be a subset of K such that:

(i) $m, m' \in M$ implies $m < m'$ or $m' < m$.

(ii) M is maximal relative to (i).

The existence of such a set follows from the Hausdorff maximality principal (Zorn's lemma).

For each $m \in M$ let $T(m) = \{m' \mid m' < m, m' \in M\}$. If $m \in M$ and $T(m)$ is not empty let $Q(m)$ be a quasi-component of $X - m$ meeting $T(m)$. If $T(m)$ is empty let $Q(m)$ be any quasi-component of $X - m$ not containing q. There is no difficulty in showing that

(a) *If $m \in M$ then $T(m) \subset Q(m)$.*

We also have

(b) *If $m' < m$ and $X - m' = Z \cup W$, $Z \mid W$, $Q(m') \subset Z$, $q \in W$, then $Q(m') \subset Z \cup m' \subset Q(m)$.*

Received July 17, 1948. Presented to the American Mathematical Society, September, 1948. This work was done under Contract N7-ONR-434, Task Order No. 3, Navy Department, the Office of Naval Research.