THE NUMBER OF POSITIVE INTEGERS $\leq x$ AND FREE OF PRIME DIVISORS $> x^c$, AND A PROBLEM OF S. S. PILLAI

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1. **Notation and conventions.** For economy of presentation and convenience in printing, the following notation and conventions are introduced at the outset.

Notation. The following symbols are used throughout the paper.

C is Euler's constant.

c, y, t are real numbers; r, n positive integers; x any real number ≥ 2, and satisfying the conditions of its context.

$[x]$ = integral part of x; $F(x) = x - [x]$.

$l = \log x$; $L = xl^{-1}$; $L_2 = xl^{-2}$.

$e\{m\} = e(m) = \exp(m)$ for every m.

$\pi(x)$ = number of primes $\leq x$; p a prime; $P(x) = \sum_{p \leq x} p^{-1}$.

$f(x, c)$ denotes the number of positive integers $\leq x$ and free of prime divisors $> x^c$.

$S(x, p)$ is the set of integers $\leq x$ each divisible by p and free of prime divisors $> p$.

$T(x, p)$ is the set of integers $\leq x$ each free of prime divisors $> p$.

$N(K)$ denotes the number of members of the set K, where K denotes any finite set of integers.

Conventions. $a_1, a_2, \cdots; b_1, b_2, \cdots; A_1, A_2, \cdots$ are positive constants each of which is chosen once and for all to suit the entire context, according as it occurs in a question or in an assertion (*viz.*, in the statement of a theorem or in the course of any proof).

2. **Introduction.** In a paper communicated elsewhere, I have proved by means of elementary theorems (*viz.*, without using the prime number theorem or any equivalent) a result which may be stated as follows.

Theorem A. *A bounded function $\phi(y)$, positive-valued for $y > 0$, and a positive-valued function $g(y)$ exist such that*

$$(1) \qquad f(x, y) = x\phi(y) + h(x, y)L; \qquad |h(x, y)| < g(y).$$

It is natural to inquire whether (1) is true with a_1 in place of $g(y)$. The affirmative answer to this question follows from the theorem of this paper which follows.

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