LAW OF TRANSFORMATION FOR ESSENTIAL GENERALIZED JACOBIANS

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Introduction.

1. Let $\mathfrak{R}$ be a bounded simply connected Jordan region in the uv-plane. Consider a continuous mapping T from $\mathfrak{R}$ into $x_1x_2x_3$-space, given by T: $x_1 = x_1(u, v)$, $x_2 = x_2(u, v)$, $x_3 = x_3(u, v)$, $(u, v) \in \mathfrak{R}$, where $x_1(u, v)$, $x_2(u, v)$, $x_3(u, v)$ are defined and continuous in $\mathfrak{R}$. Continuous flat mappings T^1, T^2, T^3 are induced by T from $\mathfrak{R}$ into the x_2x_3 -, x_3x_1 -, x_1x_2-planes respectively:

$$T^1: \quad x_1 = 0, \quad x_2 = x_2(u, v), \quad x_3 = x_3(u, v) \qquad ((u, v) \in \mathfrak{R});$$

$$T^2: \quad x_1 = x_1(u, v), \quad x_2 = 0, \quad x_3 = x_3(u, v) \qquad ((u, v) \in \mathfrak{R});$$

$$T^3: \quad x_1 = x_1(u, v), \quad x_2 = x_2(u, v), \quad x_3 = 0 \qquad ((u, v) \in \mathfrak{R}).$$

2. Introduce a new system of cartesian coordinates $x_1'x_2'x_3'$ by relations $x_i' = c_i + a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3$ for $i = 1, 2, 3$, where the c_i, a_{ij} are real constants, the matrix $\| a_{ij} \|$ is normal and orthogonal with a determinant $+1$. In terms of the new coordinates $x_1'x_2'x_3'$, the mapping T becomes

$$T': \quad x_i' = x_i'(u, v) = c_i + a_{i1}x_1(u, v) + a_{i2}x_2(u, v) + a_{i3}x_3(u, v),$$

where $(u, v) \in \mathfrak{R}$, for $i = 1, 2, 3$; and the induced flat mappings T'^1, T'^2, T'^3 are given by formulas analogous to those for T^1, T^2, T^3, respectively.

3. Now let (u, v) be any point in $\mathfrak{R}^0$ where both first partial derivatives exist for each of the functions $x_1(u, v)$, $x_2(u, v)$, $x_3(u, v)$. Thus the ordinary Jacobians are defined at (u, v) for each of the flat mappings T^1, T^2, T^3; set

$$J(u, v, T^1) = x_{2u}x_{3v} - x_{2v}x_{3u}, \qquad J(u, v, T^2) = x_{3u}x_{1v} - x_{3v}x_{1u},$$

$$J(u, v, T^3) = x_{1u}x_{2v} - x_{1v}x_{2u}.$$

Clearly both first partial derivatives exist at (u, v) for each of the functions $x_1'(u, v)$, $x_2'(u, v)$, $x_3'(u, v)$, and thus the ordinary Jacobians $J(u, v, T'^1)$, $J(u, v, T'^2)$, $J(u, v, T'^3)$ also exist. It is well known that the ordinary Jacobians transform by the law

$$J(u, v, T'^i) = a_{i1}J(u, v, T^1) + a_{i2}J(u, v, T^2) + a_{i3}J(u, v, T^3) \qquad (i = 1, 2, 3).$$

4. It is the purpose of this note to prove that the essential generalized Jacobians transform almost everywhere by the same law as the ordinary Jacobians. For a careful discussion of the essential generalized Jacobians, as

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