THE REGULARITY DOMAINS OF SOLUTIONS OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS IN TERMS OF THE SERIES DEVELOPMENT OF THE SOLUTION

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1. **Introduction.** One of the problems which has received considerable attention recently (in particular in some investigations by Bergman [3], [6]) is the question of reformulating methods of the theory of functions of a complex variable in such a way that they can be easily generalized to other fields, in particular to the theory of functions of two real variables which functions satisfy partial differential equations of elliptic type.

One of the tools used in this task is the introduction of integral operators which transform functions of one complex variable into complex solutions of a given partial differential equation

$$\mathbf{L}(u) \equiv \tfrac{1}{4}\Delta u + \tfrac{1}{2}Au_x + \tfrac{1}{2}Bu_y + Cu = 0, \tag{1.1}$$

where A, B and C are supposed to be entire functions of x and y. These complex solutions u play a role similar to that of analytic functions of a complex variable in the theory of harmonic functions. Then, by taking the real or imaginary part of u, we obtain real solutions U. In this manner various conclusions pertaining to real solutions can be obtained.

These operators (see [3], [6]) preserve various properties of the class of functions to which they are applied and therefore can be used as a successful instrument for "translating" relations and results in the theory of functions of one complex variable to the theory of solutions of (1.1).

In the present paper we show that between various subsequences of the series development of a real or a complex solution of (1.1),

$$U(z, z^*) = \sum D_{mn}z^m z^{*n}, \qquad u(z, z^*) = \sum u_{mn}z^m z^{*n}, \tag{1.2}$$

respectively, and properties of U and u, there exist various relations. Here $z = x + iy$, $z^* = x - iy$, and $D_{mn} = D^*_{nm}$. It is of interest that some of these relations are completely *independent of the coefficients* A, B and C of the partial differential equations which coefficients have only to be assumed to be entire functions of x $(= (z + z^*)/2)$ and y $(= (z - z^*)/2i)$. In some other cases these relations depend in addition only on certain *subsequences* of the coefficients $\{A_{mn}\}$, $\{B_{mn}\}$, $\{C_{mn}\}$ of the series development of

$$A = \sum A_{mn}x^m y^n, \qquad B = \sum B_{mn}x^m y^n, \qquad C = \sum C_{mn}x^m y^n.$$

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