A HOMOGENEOUS INDECOMPOSABLE PLANE CONTINUUM

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In this paper we shall answer the following question raised [2] by Knaster and Kuratowski: *If a nondegenerate bounded plane continuum is homogeneous, is it necessarily a simple closed curve?*

Mazurkiewicz gave [3] a partial solution to the above question by showing that the simple closed curve is the only homogeneous nondegenerate bounded locally connected plane continuum.

Waraszkiewicz announced [5] that the answer to the above question was in the affirmative. Choquet went even further in announcing that every homogeneous bounded closed plane set belongs to one of the following types: (1) a finite number of points; (2) a totally disconnected perfect set; (3 and 4) a set homeomorphic to the sum of a collection of concentric circles such that the common part of this sum and a line through the center of the circles is either a finite set or a totally disconnected perfect set. However, the findings of this paper do not hold with these announcements.

In this paper we shall give an example of a homogeneous nondegenerate bounded plane continuum which is not a simple closed curve.

1. **Definitions.** We give the following definitions of some terms that we shall use.

Homogeneous. A set S is homogeneous if, for each pair of its points P_1 and P_2, there is a homeomorphism that carries S into itself and P_1 into P_2. This is somewhat of a "point" condition and it might be well to call S pointwise homogeneous if these conditions are satisfied. However, we shall follow the usual terminology and call it homogeneous. Theorem 15 shows that the continuum we describe has an even stronger type of homogeneity.

Chain. A chain $D = [d_1, d_2, \cdots, d_n]$ is a collection of domains $d_1, d_2, \cdots, d_n$ such that d_i intersects d_j if and only if i is equal to $j - 1, j$, or $j + 1$. The element d_i is the i-th link of the chain D and d_1 and d_n are its end links. Links which are not end links are called interior links. Two links are adjacent if they intersect. If P and Q are points belonging to d_1 and d_n respectively but to no other link of the chain, then $D = [d_1, d_2, \cdots, d_n]$ is called a chain from P to Q. We have not supposed that the links of a chain are necessarily connected.

Contain. The chain D contains the chain E if each link of E is a subset of a link of D.

Subchain. If E is a chain each of whose links is a link of the chain D, then E is a subchain of D. If the end links of E are the i-th and j-th links of D, we may denote E by $D(i, j)$.

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