VALUE DISTRIBUTION OF A MEROMORPHIC FUNCTION OF TWO COMPLEX VARIABLES ON NON-ANALYTIC MANIFOLDS

BY JOSEPHINE MITCHELL

1.1. **Introduction.** In analogy to the geometric ideas used in the theory of functions of one complex variable the domain of definition of functions $f(z_1, z_2)$ of two complex variables $z_k = x_k + iy_k$, $k = 1, 2$, can be interpreted as a region in four dimensional Euclidean space with coordinates x_1, y_1, x_2 and y_2. However the behavior of analytic functions of two complex variables is completely different from that, say, of analytic functions of four real variables defined in the same domain. In particular, in the theory of functions of two complex variables certain two and three dimensional manifolds (analytic surfaces and hypersurfaces respectively), as well as manifolds connected in various ways with these surfaces and hypersurfaces, play an exceptional role, so that the geometry of the space of two complex variables is quite different from that of ordinary four dimensional space. In this connection various problems arise such as questions concerning the behavior of functions of two complex variables in the various manifolds indicated above. The present paper is devoted to a question of this type.

A surface $A^2 = \mathrm{E}[z_k = g_k(Z)]$, $k = 1, 2$, where g_k are analytic functions of a single complex variable Z, is called an analytic surface. In A^2, f becomes an analytic function $F(Z) \equiv f[g_1(Z), g_2(Z)]$ of the single variable Z and therefore has all the properties of such functions. A family of analytic surfaces $h^3 = \mathrm{E}[z_k = h_k(Z, \lambda)]$, $k = 1, 2$, where h_k are continuous functions of Z and a real variable λ, which for every fixed λ are analytic functions of Z, forms an analytic hypersurface.

Analytic surfaces and hypersurfaces have been considered for some time but only recently Bergman has indicated new methods by which their properties may be utilized in order to obtain tools for the investigation of functions $f(z_1, z_2)$. He introduced domains M^4 whose boundary m^3 consists of a finite number of segments of analytic hypersurfaces h_k^3, $k = 1, \cdots, N$. (See [1].) Let $\dot{i}_k^3$ denote the segment of h_k^3 which belongs to the boundary $m^3 = S_{k=1}^N \dot{i}_k^3$ of M^4. The sum of the intersections of these hypersurfaces, $S_{j,k=1}^N \dot{i}_j^3 \cap \dot{i}_k^3$, $k \neq j$, forms a two dimensional manifold F^2, the distinguished boundary surface of M^4. (A similar situation arises in the case of a three dimensional domain bounded by a finite number of planes, that is, a polyhedron—the sum of the edges of the polyhedron forms a one dimensional domain.) In spite of the fact that the distinguished boundary surface represents only a part of the

Received February 3, 1947; presented to the Society August 23, 1946. The author wishes to thank S. Bergman for his many helpful suggestions and criticisms given during the preparation of this paper.