THE ADDITIVE PROPERTIES OF INTEGERS OF A CERTAIN CLASS

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Evelyn and Linfoot [3], [4], [5], [6], [7] obtained an asymptotic formula for the number of representations of a large integer n as the sum of s r-free integers, and their results were later sharpened by Barham and Estermann [1] and by me [8]. (If $r \geq 2$, an integer is called r-free if it is not divisible by the r-th power of any prime.) In the present note I shall be concerned with a more general problem. The methods I use are different from those introduced by the other authors in dealing with the original problem of r-free integers, though the argument in §2 owes something to a paper by Estermann [2].

Let **A** be any given class (finite or infinite) of integers greater than 1, and such that any two integers belonging to it are coprime. Members of **A** will be called *a-numbers*, and the letter a will be reserved for them. A number will be called **A**-*free* if it is not divisible by any a-number. For $s \geq 2$ we shall denote by $Q(n) = Q(n, \mathbf{A}, s)$ the number of representations of n (order being relevant) as the sum of s **A**-free numbers. Our object is to investigate the behavior of $Q(n)$ as $n \to \infty$.

It will be assumed throughout §§1–3 that the series

$$\sum_{a} 1/a \tag{1}$$

converges, and in §1 I shall obtain an asymptotic formula for $Q(n)$. If, furthermore, (1) converges sufficiently rapidly (*i.e.*, if the frequency of a-numbers is not too great), the error term in this formula can be sharpened considerably; this sharpening will be effected in §2. In §3 I shall investigate the average order of the error term in the asymptotic formula for $Q(n)$. Finally in §4 the case when (1) diverges will be briefly considered. The problem is then naturally much more difficult, and I am at present only able to obtain a rather inadequate upper estimate for $Q(n)$.

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1. **Notation.** Let $P_1(\xi)$ and $P_2(\xi)$ be two propositions concerning a variable ξ. Then

$$P_1(\xi) \qquad (P_2(\xi))$$

means that for *every* ξ for which $P_2(\xi)$ holds, $P_1(\xi)$ holds also;

$$P_1(\xi) \qquad [P_2(\xi)]$$

means that for *some* ξ for which $P_2(\xi)$ holds, $P_1(\xi)$ holds also.

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