# LIE GROUPS OF GENUS ONE

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1. **Introduction.** The genus of a Lie group has been defined by M. S. Knebelman [2] as the difference between its order and nullity, where the nullity of the group is the least number of independent symbols whose successive commutators generate the group. We shall denote a group of order $n$ and genus $r$ by $G_n^{(r)}$.

It is shown in [2] that the constants of structure $c^i_{jk}$ of a $G_n^{(r)}$ must be of the form

$$(1.1)\qquad c^i_{jk} = \sum_{\lambda=1}^{r} p^{\lambda}_{jk} s^i_{\lambda} + \delta^i_j u_k - \delta^i_k u_j \qquad (r < n - 2),$$

where $p^{\lambda}_{jk}$ are a set of $r$ skew-symmetric tensors and $s^i_{\lambda}$ are a set of $r$ contravariant vectors such that the matrices $\| p^{\lambda}_{jk} \|$, $\| s^i_{\lambda} \|$ are each of rank $r$.

Necessary and sufficient conditions on the $c^i_{jk}$ were obtained in [2] in order that a group be of genus 0. These conditions are expressed as

$$(1.2)\qquad \Pi^i_{jk} \equiv c^i_{jk} - (n-1)^{-1}(\delta^i_j c^h_{hk} - \delta^i_k c^h_{hj}) = 0.$$

Also certain analogies between $\Pi^i_{jk}$ and the projective connection of an affine space were noted.

In this paper we obtain necessary and sufficient conditions on the $c^i_{jk}$ for the group to be of genus 1. As part of these conditions a comitant $W^h_{ijk}$ is introduced which is analogous to the Weyl projective curvature tensor. For a $G_n^{(0)}$, $W^h_{ijk} \equiv 0$.

2. **Necessary conditions.** If $r = 1$ we must have $n > 3$ due to the condition $r < n - 2$, and this condition on $n$ is assumed in all summations. Equations (1.1) can then be expressed as

$$(2.1)\qquad c^i_{jk} = p_{jk} s^i + \delta^i_j u_k - \delta^i_k u_j .$$

The necessary conditions are obtained by elimination of $p_{jk}$, $s^i$, $u_i$ from (2.1).

It is shown in [2] that if $u_i$ is not a zero vector,

$$(2.2)\qquad p_{ij} s^i = N u_j ,$$

$$(2.3)\qquad (N-1)(p_{ij} u_k + p_{jk} u_i + p_{ki} u_j) = 0,$$

$$(2.4)\qquad s^i u_i = 0,$$

where $N$ is an arbitrary constant. Equations (2.2)–(2.4) are in fact equivalent to Jacobi's identities

$$(2.5)\qquad c^h_{jk} c^i_{hm} + c^h_{km} c^i_{hj} + c^h_{mj} c^i_{hk} = 0.$$

Received December 11, 1947.