ASYMPTOTIC INTEGRATIONS OF THE ADIABATIC OSCILLATOR IN ITS HYPERBOLIC RANGE

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Let $f(t)$ be a real-valued, continuous function defined for large positive t in such a way that $f(\infty)$ exists (as a finite limit). In [4], various general results were developed for the asymptotic integration of the corresponding linear differential equation

$$x'' + f(t)x = 0 \tag{1}$$

for the "elliptic" case, $f(\infty) > 0$, where (1) is for large t of the type $x'' + x = 0$. In the present paper, a corresponding theory will be developed for the "hyperbolic" case, $f(\infty) < 0$; so that, if the unit of length on the t-axis is so chosen that $f(\infty) = -1$, the differential equation (1) can be written in the form

$$x'' - (1 + \phi)x = 0, \tag{2}$$

where

$$\phi(t) \to 0 \qquad (t \to \infty). \tag{3}$$

Whereas the results will be parallel to those obtained in [4] for the elliptic case, the proofs and the tools needed will be substantially different. The discrepancy is not surprising, since the energy considerations which, in the elliptic case, lead to formal stability are not now available and must be replaced by exponential approximations.

Part I.

1. In order to avoid an interruption of the treatment of (2) in the case (3), a general Abelian lemma and its variants will first be isolated.

(I) *If* $p(t)$, $q(t)$, *where* $0 \leq t < \infty$, *are two continuous* (*possibly complex-valued*) *functions satisfying*

$$p(t) \neq 0 \tag{4}$$

and

$$\int_0^\infty \int_0^t |\, q(s) \,|\, ds / \,|\, p(t) \,|\, dt < \infty, \tag{5}$$

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