SKEWLY CEVIAN TETRAHEDRONS

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1. Intersection of a quadric with a tetrahedron.

a. Theorem. *If six pairs of points marked on the edges of a tetrahedron are such that the six points lying in each of three faces of the tetrahedron belong to a conic,* (i) *the same is true of the points lying in the fourth face, and* (ii) *the twelve points lie on a quadric surface.*

Let X', X''; Y', Y''; Z', Z''; U', U''; V', V''; W', W'' be pairs of points situated on the edges BC, CA, AB, DA, DB, DC, of a tetrahedron $(T) = ABCD$, and suppose that the points in each of the faces DBC, DCA, DAB, lie on a conic.

The nine points U', U''; V', V''; W', W''; X', Y', Z', no three of which are collinear and no six coplanar, determine a quadric (Q). The six points in the face DBC lie on a conic, by assumption, and five of them lie on (Q), by construction, hence the sixth point, X'', also lies on (Q). Similarly for Y'', Z'', in the faces DCA, DAB. Hence all the twelve points lie on (Q), and therefore the points in the face ABC also lie on a conic. Hence the proposition.

Otherwise. The two conics in the two faces DAB, DAC have a pair of points U', U'' in common, hence the two conics lie on an infinite number of quadric surfaces forming a pencil whose base is the degenerate skew quartic curve formed by the two conics. Let (Q) be the quadric of the pencil which passes through the point X'. The plane DBC cuts (Q) along a conic which has five points in common with the given conic in that plane, hence the sixth point X'' also lies on both conics and therefore lies on (Q).

b. As an obvious consequence we have the

Theorem. *On three concurrent edges DA, DB, DC, of a tetrahedron $(T) = ABCD$ are marked three pairs of points U', U''; V', V''; W', W''. Three arbitrary conics drawn through the three tetrads of points U', U'', V', V''; V', V'', W', W''; W', W'', U', U'', meet the third edge of the respective face of (T) in three pairs of points which lie on a conic.*

c. It may be of some interest to observe that this proposition is an extension, or a generalization of Desargues' theorem on perspective triangles. Indeed, among the conics passing through the four points U', U'', V', V'' we may consider the degenerate conic constituted by the lines $U'V'$ and $U''V''$; let Z', Z'' be their respective traces on the edge AB. Similarly let the lines $V'W'$, $V''W''$ meet BC in X', X'', and the lines $W'U'$, $W''U''$ meet CA in Y', Y''. The conic on which the six points lie is in the present case degenerate, for the points X', Y', Z' lie on the trace of the plane $U'V'W'$ in the plane ABC, and the points X'', Y'', Z'' lie on the line of intersection of the planes $U''V''W''$,

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