SUMMABILITY FACTORS OF FOURIER SERIES AT A GIVEN POINT

By Min-Teh Cheng

1. Let $f(t)$ be a summable function, periodic with period 2π. Let the Fourier series of $f(t)$ be

$$\tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) \equiv \sum_{n=0}^{\infty} A_n(t).$$

We write

$$\phi(t) = \tfrac{1}{2}\{f(x + t) + f(x - t)\},$$

$$\Phi_\alpha(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - u)^{\alpha-1}\phi(u)\, du \qquad (\alpha > 0),$$

$$\phi_\alpha(t) = \Gamma(\alpha + 1)t^{-\alpha}\Phi_\alpha(t) \qquad (\alpha > 0),$$

and

$$\Phi_0(t) = \phi_0(t) = \phi(t).$$

A series $\sum a_n$ is said to be absolutely summable (C, α), or summable $| C, \alpha |$, if the series

$$\sum | \sigma_n^\alpha - \sigma_{n-1}^\alpha |$$

converges, where σ_n^α denotes the n-th Cesàro mean of order α of the series $\sum a_n$, *i.e.*,

$$\sigma_n^\alpha = \frac{1}{(\alpha)_n} \sum_{\nu=0}^{n} (\alpha)_{n-\nu} a_\nu\,, \qquad (\alpha)_\nu = \frac{\Gamma(\alpha + \nu + 1)}{\Gamma(\alpha + 1)\Gamma(\nu + 1)} \qquad (\alpha > -1).$$

L. S. Bosanquet [2] proved that, if $\phi_\alpha(t)$ is of bounded variation in $(0, \pi)$, then the Fourier series of $f(t)$ is summable $| C, \beta |$ at the point $t = x$ for $\beta > \alpha \geq 0$. In the present note the following theorem is established.

Theorem. *If $\phi_\alpha(t)$, $0 \leq \alpha \leq 1$, is of bounded variation in $(0, \pi)$, then the series*

$$\sum \frac{A_n(t)}{(\log n)^{1+\epsilon}} \qquad (\epsilon > 0)$$

is summable $| C, \alpha |$ at the point $t = x$.

Received March 4, 1947.