THE TYPE OF THE POLYNOMIALS GENERATED BY $f(xt)\phi(t)$

By William N. Huff

1. **Introduction.** We wish to consider polynomial sets $\{y_n(x)\}$ defined by the generating function

$$(1.1) \qquad g(x, t) = f(xt)\phi(t) = \sum_{n=0}^{\infty} y_n(x)t^n,$$

where

$$f(xt) = \sum_{n=0}^{\infty} a_n \frac{(xt)^n}{n!}, \qquad \phi(t) = \sum_{n=0}^{\infty} b_n \frac{t^n}{n!}.$$

The notation $\{y_n(x)\}$ for a set of polynomials always implies a generating function of form (1.1). We assume $b_0 \neq 0$, $a_n \neq 0$ for all n, and $y_{-i}(x) \equiv 0$ for $i = 1, 2, 3, \cdots$.

Using a notation similar to that of W. C. Brenke [1] the polynomials $y_n(x)$ may be written in the symbolic form

$$(1.2) \qquad y_n(x) = \frac{1}{n!}(b + ax)^{(n)},$$

where b^i and a^i are replaced by b_i and a_i after expansion.

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2. **Polynomials of A-type.** Every polynomial set $\{P_n(x)\}$ where $P_n(x)$ is of degree n satisfies a unique relation of the form

$$(2.1) \qquad P_{r-1}(x) = \sum_{n=1}^{\infty} L_n(x)P_r^{(n)}(x) = \sum_{n=1}^{\infty} (l_{n0} + l_{n1}\, x + \cdots + l_{n,n-1}x^{n-1})P_r^{(n)}(x),$$

with $nl_{10} + n(n-1)l_{21} + \cdots + n!l_{n,n-1} \neq 0$ for $n = 1, 2, 3, \cdots$.

Definition. If no coefficient $L_n(x)$ is of degree exceeding k but at least one $L_n(x)$ is of degree k, the set $\{P_n(x)\}$ is said to be of A-type k. If the degrees of the coefficients $L_n(x)$ are unbounded, the set $\{P_n(x)\}$ is of infinite A-type.

The above result and definition are due to I. M. Sheffer [3].

If the coefficients $L_n(x)$ are constants, the set of polynomials $\{P_n(x)\}$ is said to be of A-type zero. The Appell and Newton polynomials each constitute such a set. The Hermite polynomials, and the Laguerre polynomials given by the generating function

$$e^{-xt/(1-t)}\left(\frac{1}{1-t}\right) = \sum_{n=0}^{\infty} L_n(x)t^n,$$

are each an orthogonal set of type zero.

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