SOME RESULTS FOR DIRICHLET SERIES

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1. **Introduction.** This paper may be regarded as an attempt to extend to Dirichlet series some results proved for Taylor series by the author [1]. These results concern the nature of domains in which a function defined by a Taylor series is regular.

2. We shall be concerned with the ordinary Dirichlet series

$$f(z) = \sum_{n=1}^{\infty} a_n e^{-z \log n}.$$

Let $\sigma_c < \infty$ denote the abscissa of convergence of this series. Suppose $l - 1 < h < l$, where l is integral and positive but otherwise arbitrary. Denote by D the domain in the w-plane, $\psi_2 \leq \arg (w - h) \leq \psi_1$, including the point $w = h$, where $0 < \psi_1 \leq \pi/2$ and $-\pi/2 \leq \psi_2 < 0$. Suppose $a(w)$ is a function regular in D, with the possible exception of the point at infinity, for which $a(n) = a_n$, $n = 0, 1, \cdots$.

Denote respectively by I_1, I_2 and C the sides of a contour Γ bounded by the lines $w = h + Re^{i\psi_1}$ and $w = h + Re^{i\psi_2}$, $0 \leq R \leq R'$, and the arc $\psi_2 \leq \arg (w - h) \leq \psi_1$ of the circle $| w - h | = R'$ where R' is integral and positive but otherwise arbitrary. It is then simple to show by the Calculus of Residues that (integration being understood in the positive sense)

$$\frac{1}{2\pi i} \int_\Gamma \frac{a(w) \exp(-z \log w)}{\exp(2\pi i w) - 1}\, dw = \sum_{n=l}^{R'+l-1} a(n) e^{-z \log n}.$$

Here we take that branch of $\log w$ that is real for w real and positive. A similar integral was used by LeRoy and Lindelöf [2] in the case of Taylor series.

Suppose for $w = h + Re^{i\psi}$ in D and $R \geq R_1$ that

$$| a(h + Re^{i\psi}) | < R^K \exp(-LR \sin \psi) \tag{1}$$

for some K and some $0 < L < 2\pi$.

Let

$$G(z, w) = \frac{a(w) \exp(-z \log w)}{\exp(2\pi i w) - 1}. \tag{2}$$

We proceed to show that

$$\int_c G(z, w)\, dw$$

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