ORDER OF MAGNITUDE OF THE ZEROS OF POLYNOMIALS IN BASIC SERIES

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1. **Introduction.** Polynomials $p_0(z)$, $p_1(z)$, $\cdots$ are said to form a basic set if they are such that every polynomial can be expressed in one and only one way as a finite linear combination of them. J. M. Whittaker [3] has shown that associated with a basic set there is a basic series,

$$p_0(z)\Pi_0 f(0) + p_1(z)\Pi_1 f(0) + \cdots . \tag{1.1}$$

If (1.1) converges uniformly to $f(z)$ in $|z| \le R$, then it is said that the basic series represents $f(z)$ in $|z| \le R$.

Let the equations expressing $1, z, z^2, \cdots$ in terms of the given polynomials be represented in general by

$$z^n = \pi_{n0}p_0(z) + \pi_{n1}p_1(z) + \pi_{n2}p_2(z) + \cdots ; \tag{1.2}$$

then

$$\Pi_n f(0) = \pi_{0n} f(0) + \pi_{1n}\frac{f'(0)}{1!} + \pi_{2n}\frac{f''(0)}{2!} + \cdots .$$

The two main theorems governing the convergence properties of a basic series are due to Whittaker [3] and Cannon [1].

Definition 1. (See [1; 10].) Let

$$R^n \le |\pi_{n0}| M_0(R) + |\pi_{n1}| M_1(R) + |\pi_{n2}| M_2(R) + \cdots = \omega_n(R),$$

where

$$M_i(R) = \max_{|z|=R} |p_i(z)|.$$

The order of a basic set of polynomials is

$$\omega = \lim_{R\to\infty} \overline{\lim_{n\to\infty}} \log \omega_n(R)/n \log n.$$

If $0 < \omega < \infty$ the type is

$$\gamma = \lim_{R\to\infty} \overline{\lim_{n\to\infty}} \frac{e}{n\omega} \{\omega_n(R)\}^{1/n\omega}.$$

Lemma 1. (See [1; 13].) *If $\{p_n(z)\}$ is a basic set of order ω, type γ and $f(z)$ is an integral function of increase less than order $1/\omega$, type $1/\gamma$, the basic series converges absolutely and uniformly to $f(z)$ in any finite region of the plane. The limitation on $f(z)$ means that its order ρ and its type σ satisfy either* (i) $\rho < 1/\omega$, *or* (ii) $\rho = 1/\omega$, $\sigma < 1/\gamma$.

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