THE SUMMABILITY (A) OF THE CONJUGATE SERIES OF A FOURIER SERIES

By M. L. Misra

1. Let the Fourier series, associated with a function $f(x)$ which is integrable (L) in $(-\pi, \pi)$ and defined outside this interval by periodicity, be

$$\frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx). \tag{1.1}$$

The conjugate series of this Fourier series is

$$\sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx). \tag{1.2}$$

The conjugate function associated with the series (1.2) is

$$g(x) = \frac{1}{2\pi} \int_0^{\pi} \psi(t) \cot \frac{1}{2} t \, dt = \frac{1}{\pi} \int_0^{\infty} \frac{\psi(t)}{t} \, dt,$$

where $\psi(t) = f(x + t) - f(x - t)$, the integrals being Cauchy integrals. Let

$$\psi_1(t) = t^{-1} \int_0^t \psi(t) \, dt, \qquad \psi_2(t) = t^{-1} \int_0^t \psi_1(t) \, dt,$$

and generally

$$\psi_n(t) = t^{-1} \int_0^t \psi_{n-1}(t) \, dt,$$

n being a positive integer.

The summability (A) of the conjugate series has been discussed by Fatou [1], Lichtenstein [3], Plessner [4], and Prasad [5]. If for $0 \leq r < 1$,

$$V(r, x) = \sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx) r^n, \qquad \epsilon = \arcsin (1 - r),$$

then Plessner proved that

$$\lim_{r \to 1} \left[V(r, x) - \frac{1}{2\pi} \int_{\epsilon}^{\pi} \psi(t) \cot \frac{1}{2} t \, dt \right] = 0,$$

provided that

$$\psi_1(t) = t^{-1} \int_0^t \psi(t) \, dt = o(1),$$

Received April 15, 1947.