REGULAR TRANSFORMATIONS ON GENERALIZED MANIFOLDS

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In a previous paper [5] the author has considered the action of an n-regular transformation (see Definition 2 below) on a compactum. In the present paper a further study is made in which the action of these transformations on generalized manifolds is considered. It is found that the regularity of the transformation is not sufficient to insure that a manifold of given dimension be transformed into one of the same dimension. However, the addition of one more restriction on the point inverses is found to insure the manifold invariance.

As in the first paper we will deal only with transformations $T(K) = K'$ where T is single-valued and continuous and K and K' are compacta. All of the ordinary complexes and cycles used shall have modulo two coefficients and the Vietoris cycles [4] used shall consist of these as coordinate cycles. Hereafter we shall refer to them simply as V-cycles. The boundary of the n-dimensional V-cycle V^n will be denoted by ∂V^n. Finally we shall denote by $S(A, \epsilon)$ the set of all points x of the space such that $\rho(y, x) < \epsilon$ for some point $y \in A$.

DEFINITION 1. The sequence of closed sets (A_i), contained in a compactum, is said to converge n-regularly to A if $\lim A_i = A$, and if corresponding to every $\epsilon > 0$ there is a $\delta > 0$ and an N such that if V^r is an r-dimensional V-cycle of A_i of diameter $<\delta$ for $i > N$ and $r \leq n$, then $V^r \sim 0$ in a subset of A_i with diameter $<\epsilon$.

DEFINITION 2. The transformation $T(K) = K'$ (where K, K' will always be compacta) will be called n-regular, provided that for every sequence of points (y_i) converging to y in K', we have that $(T^{-1}(y_i))$ converges n-regularly to $T^{-1}(y)$ in K.

(It follows from the Eilenberg characterization of interior transformations [3] that an n-regular transformation is interior.)

The main results of the previous paper followed from two lemmas which will also be used in this work; they are stated below for future reference.

LEMMA A. *If $T(K) = K'$ is $(n - 1)$-regular, then for any $e > 0$ there exists a $d > 0$ such that for any n-dimensional V-cycle $(V^n)'$ of K' with diameter $<d$, there exists a V-cycle V^n of K with diameter $<e$ and such that $T(V^n) = (V^n)'$.*

This is Theorem 2.5 of [5].

LEMMA B. *If $T(K) = K'$ is n-regular, then for any $e > 0$ there exist numbers $d > 0$, $d' > 0$ such that every r-dimensional V-cycle V^r $(r \leq n)$ of K with diameter $<d'$ such that $T(V^r) \sim 0$ in a subset of K' with diameter $<d$ is itself ~ 0 in a subset of K with diameter $<e$.*

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