THE ORTHOGONALITY OF SOME SYSTEMS OF POLYNOMIALS

BY J. GERONIMUS

Introduction. Consider a system of polynomials $\{P_n(x)\}_0^\infty$ where

$$P_n(x) = \sum_{\nu=0}^{n} a_{n-\nu} b_\nu \omega_\nu(x), \qquad \omega_\nu(x) = \prod_{k=1}^{\nu} (x - x_k) \tag{1}$$

$$(\nu = 1, \cdots, n; n = 1, 2, \cdots),$$

$$P_0 = a_0 b_0,$$

$\{a_k\}_0^\infty$, $\{b_k\}_0^\infty \neq 0$ and $\{x_k\}_1^\infty$ being arbitrary complex numbers. Some types of such systems have been considered by different authors as the table shows.

N	x_n	a_n	b_n	Author	Note
I	0	a_n	b_n	Angelesco [1]	a_n, b_n arbitrary
II	0	$\frac{\alpha_n}{n!}$	$\frac{1}{n!}$	Appell [3]	α_n arbitrary
III	$n - 1$	$\frac{1}{n!}$	$\frac{(1 - e^\lambda)^n}{n!^2}$	Gottlieb [8]	
IV	$n - 1$	$\frac{1}{n!}$	$\frac{(1 - e^\lambda)^n}{n!\Gamma(\alpha + n + 1)}$	Feldheim [5]	
V	$n - 1$	$\frac{(-1)^n}{n!}$	$\frac{a^{-n}}{n!}$	Poisson-Charlier [12]	
VI	$n - 1$	$\frac{(-a)^n}{n!}$	$\frac{1}{n!}$	Jordan [9], Geronimus [6]	
VII	0	$\binom{\lambda}{n}$	$\binom{\mu}{n} h^n$	Angelesco [2]	
VIII	0	$\frac{(-\lambda)^n}{\mu_n}$	$\frac{1}{\mu_n}$	Bateman [4]	$\mu_n = \prod_{i=1}^{n} (1 - q^i)$ $(\lvert q \rvert < 1)$
IX	0	$\frac{1}{\mu_n}$	$\frac{(-1)^n q^{n^2+n/2}}{\mu_n}$	Stieltjes [11], Wigert [13]	$\mu_n = \prod_{i=1}^{n} (1 - q^i)$ $(\lvert q \rvert < 1)$

Received February 14, 1947.