SUMMABILITY FACTORS OF FOURIER SERIES AT A GIVEN POINT

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1. Let $f(x)$ be integrable in the sense of Lebesgue and periodic with period 2π. Let the Fourier series of $f(x)$ be

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \equiv \sum_{n=0}^{\infty} A_n(x). \tag{1.1}$$

We write

$$\phi(t) = \phi_x(t) = \tfrac{1}{2}\{f(x+t) + f(x-t) - 2f(x)\}. \tag{1.2}$$

A series $\sum a_n$ is said to be absolutely summable (C, α), $\alpha > -1$, or $| C, \alpha |$, if the series

$$\sum | \sigma_n^\alpha - \sigma_{n-1}^\alpha |$$

converges, where σ_n^α denotes the n-th Cesàro mean of order α of the series $\sum a_n$, *i.e.*,

$$\sigma_n^\alpha = \frac{1}{(n)_\alpha} \sum_{\nu=0}^{n} (n - \nu)_\alpha a_\nu , \qquad (n)_\alpha = \frac{\Gamma(\alpha + n + 1)}{\Gamma(\alpha + 1)\Gamma(n + 1)}.$$

In the present paper we shall consider the summability factors of (1.1) at a given point x such that

$$\int_0^t | \phi_x(u) | \, du = O\left(t \left(\log \frac{1}{t} \right)^\beta \right) \qquad (\beta \geq 0) \tag{1.3}$$

holds. (See [5], [3].) The following theorems are established.

Theorem 1. *The series*

$$\sum \frac{A_n(x)}{(\log n)^{1+\beta+\epsilon}}$$

is summable $| C, \alpha |$, $\alpha > 1$, *at the given point* x.

Theorem 2. *The series*

$$\sum \frac{A_n(x)}{(\log n)^{1+\frac{1}{2}+\beta+\epsilon}}$$

is summable $| C, 1 |$ *at the given point* x.

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