SUMS OF AN EVEN NUMBER OF SQUARES IN $GF[p^n, x]$

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1. **Introduction.** Let n be an arbitrary positive integer and p a positive (odd) prime. Then $GF(p^n)$ denotes the Galois field of order p^n and $GF[p^n, x]$ the ring of polynomials in an indeterminate x with coefficients in $GF(p^n)$. The purpose of this paper is to find the number of representations of a polynomial of $GF[p^n, x]$ as a sum of an even number of squares, subject to certain mild restrictions.

Let $\alpha_1, \cdots, \alpha_{2s}$ be $2s$ non-zero elements of $GF(p^n)$ and place

$$\epsilon = \alpha_1 + \cdots + \alpha_{2s} . \tag{1.1}$$

By a primary polynomial we mean a polynomial of $GF[p^n, x]$ in which the coefficient of the highest power of x is the unit element of the field.

The problem under consideration will be divided into two parts:

I. Suppose F is primary of even degree $2k$ and $\epsilon \neq 0$. Then we want the number of solutions of

$$\epsilon F = \alpha_1 X_1^2 + \cdots + \alpha_{2s} X_{2s}^2 \tag{1.2}$$

in primary polynomials X_i of degree k. If F is arbitrary of degree less than $2k$ and if $\epsilon = 0$, then we want the number of solutions of

$$F = \alpha_1 X_1^2 + \cdots + \alpha_{2s} X_{2s}^2 \tag{1.3}$$

in primary polynomials of degree k.

II. Suppose F is primary of degree $2k$, m is an integer such that $2s > m \geq 1$, and $\beta \neq 0$ where $\beta = \alpha_1 + \cdots + \alpha_m$. We want the number of solutions of

$$\beta F = \alpha_1 X_1^2 + \cdots + \alpha_{2s} X_{2s}^2 , \tag{1.4}$$

where $X_1, \cdots, X_m$ are primary of degree k and $X_{m+1}, \cdots, X_{2s}$ are arbitrary of degree less than k. On the other hand, if $\beta = 0$ and F is arbitrary of degree less than $2k$, we want the number of solutions of

$$F = \alpha_1 X_1^2 + \cdots + \alpha_{2s} X_{2s}^2 \tag{1.5}$$

where $X_1, \cdots, X_m$ are primary of degree k and $X_{m+1}, \cdots, X_{2s}$ are arbitrary of degree less than k.

Suppose M is an arbitrary polynomial of $GF[p^n, x]$. Using deg for degree, we define (see [3])

$$\delta_z(M) = \begin{cases} \displaystyle\sum_{Z \mid M}^{\deg Z = z} 1 & (z \geq 0) \\ 0 & (z < 0), \end{cases} \tag{1.6}$$

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