# THE SUMMABILITY (A) OF THE SUCCESSIVELY DERIVED SERIES OF A FOURIER SERIES AND ITS CONJUGATE SERIES

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1. Let $f(\theta)$ be a function which is integrable $(L)$ in $(-\pi, \pi)$ and defined outside this range by periodicity with a period $2\pi$. Let the Fourier series of $f(\theta)$ be

$$\tag{1.1} \tfrac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta).$$

Then the conjugate series of this Fourier series is

$$\tag{1.2} \sum_{n=1}^{\infty} (b_n \cos n\theta - a_n \sin n\theta).$$

The series

$$\sum_{n=1}^{\infty} \frac{d^r}{d\theta^r} (a_n \cos n\theta + b_n \sin n\theta),$$

$$\sum_{n=1}^{\infty} \frac{d^r}{d\theta^r} (b_n \cos n\theta - a_n \sin n\theta)$$

are respectively the $r$-th derived series of the Fourier series (1.1) and its conjugate series (1.2).

We know that this function $f(\theta)$ possesses, at the point $\theta$, the $r$-th generalized symmetric derivative denoted by $f_{(r)}(\theta)$, in the sense of de la Vallee-Poussin, if $f(\theta)$ admits, for small values of $t$, developments of the form

$$\tfrac{1}{2} [f(\theta + t) + f(\theta - t)] = f(\theta) + \frac{t^2}{2!} f_{(2)}(\theta) + \cdots + \frac{t^{2k-2}}{(2k-2)!} f_{(2k-2)}(\theta) + [f_{(2k)}(\theta) + \epsilon_t] \frac{t^{2k}}{(2k)!},$$

for $r = 2k$, $k$ being a positive integer, and

$$\tfrac{1}{2} [f(\theta + t) - f(\theta - t)] = t f_{(1)}(\theta) + \frac{t^3}{3!} f_{(3)}(\theta) + \cdots + \frac{t^{2k-1}}{(2k-1)!} f_{(2k-1)}(\theta) + [f_{(2k+1)}(\theta) + \epsilon_t] \frac{t^{2k+1}}{(2k+1)!}$$

for $r = 2k + 1$, where $\epsilon_t \to 0$ as $t \to 0$.

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