SOME THEOREMS IN DIMENSION THEORY FOR NORMAL HAUSDORFF SPACES

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1. **Introduction.** The purpose of this paper is to prove for normal Hausdorff spaces a number of theorems which are known for separable metric spaces. The equivalence of Lebesgue's covering definition in terms of open sets to the definition of Alexandroff in terms of mappings into the n-simplex is proved together with some other equivalences. Another proof of the sum theorem for countably many closed sets is given; and it is proved that the dimension of the Cartesian product of two compact spaces does not exceed the sum of the dimensions of the spaces.

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2. **Notation and Definitions.** The operations of the algebra of sets will be written as in Kuratowski [9]. In particular, lower case Roman subscripts will be used only when the range of the subscripts is finite or countable.

A *Hausdorff space* is a T_2 space in the sense of Alexandroff and Hopf. A space is called normal if all pairs of disjoint closed sets have disjoint neighborhoods.

Simplices and complexes will be used only in their most elementary geometrical sense. The *n-simplex,* written σ^n, is defined as the convex hull of any set of $n + 1$ independent points in Euclidean n-space. The $n + 1$ points will be called the *vertices* of σ^n. The *boundary* of σ^n, written S^{n-1}, is the sum of the convex hulls of all sets of n vertices of σ^n; these $(n - 1)$-simplices will be called the *faces* of σ^n. A *complex*, K, will be a finite collection of simplices with the property that if $\sigma^n \varepsilon K$ then each face of σ^n is an element of K and with the additional property that two simplices can intersect only in a simplex of K. In a complex the *open star with center at a vertex* is the complement of the set of simplices in the complex which do not contain the vertex.

All the usual properties of simplices will be assumed. In particular, the dimension of the n-simplex, in any sense of the word dimension, is n; and the dimension of the Cartesian product of two simplices is the sum of their dimensions. Sets homeomorphic to the n-simplex in which it is desired to have other than simplicial subdivisions of the boundary will be called n-cells. The set S^n will be called the *n-sphere.* Continuous functions will be called *mappings.* If f is a mapping, $f : X \rightarrow Y$, and $A \subset X$, then by f/A is meant the mapping obtained by restricting the domain of f to A. A mapping $f : X \rightarrow S^n$ will be called

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