SOME PROPERTIES OF SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS GIVEN BY THEIR SERIES DEVELOPMENT

By Josephine Mitchell

1. **Introduction.** Any analytic function of two real variables, $U(x, y)$, is completely determined by the coefficients of the Taylor series $\sum_{m,n=0}^{\infty} B_{mn}x^m y^n$ of the function at the origin. For convenience we may introduce the complex variables $z = x + iy$, $z^* = x - iy$ and write

$$(1.1) \qquad U(z,z^*) = \sum_{m,n=0}^{\infty} D_{mn}z^m z^{*n}, \qquad D_{mn} = D_{nm}^* \qquad (m, n = 0, 1, 2, \cdots).$$

Then U is completely determined by the coefficients D_{mn}, $m \leq n$. (The symbol U stands for either $U(x, y)$ or $U(z, z^*)$ depending on the context. A star as superscript usually means the conjugate $z^* = x - iy$ of $z = x + iy$, although z and z^* may sometimes be treated as independent complex variables as is done for example in equation (2.3).) It is clear that some relations must exist between the properties of the function U and those of the coefficients D_{mn} which determine it, even though no such relations are known at present.

If harmonic functions, $h(x, y)$, of two real variables are considered, such relationships are known and are comparatively simple. The harmonic function $h(x, y)$ can be regarded as the real part of an analytic function $f(z)$, and if $f(z)$ has the series development $\sum_{m=0}^{\infty} a_m z^m$, then $h(x, y) = \sum_{m=0}^{\infty} \frac{1}{2}(a_m z^m + a_m^* z^{*m})$ or equals $\sum_{m=0}^{\infty} (D_{m0}z^m + D_{0m}z^{*m})$, where $D_{m0} = \frac{1}{2}a_m$, $D_{0m} = D_{m0}^*$. In this case all D_{mn} with m or n not equal to 0 are zero and the double sequence of coefficients has reduced to the sequence $\{D_{m0}\}$. Therefore the subsequence $\{D_{m0}\}$ completely determines all properties of the function h.

The simplification obtained in the case of functions which satisfy $\Delta U = 0$ suggests the study of other special classes of analytic functions of two real variables such as those which satisfy the linear partial differential equation

$$\begin{aligned} \mathbf{L}_1(U) &\equiv \Delta U + A(x, y)U_x + B(x, y)U_y + C(x, y)U \\ (1.2.\text{I}) \qquad &\equiv \frac{4\partial^2 U}{\partial z\, \partial z^*} + 2\text{Re}\left[\left(\sum_{m,n=0}^{\infty} a_{mn}z^m z^{*n}\right)\frac{\partial U}{\partial z}\right] + \left(\sum_{m,n=0}^{\infty} c_{mn}z^m z^{*n}\right)U \\ &\equiv 4U_{zz^*} + 2aU_z + 2a^*U_{z^*} + cU = 0, \end{aligned}$$

where a and c are entire functions of two real variables. (Referred to as Case I in this paper.) *In this case if a_{mn} and c_{mn} $(m, n = 0, 1, \cdots)$ are given,* it is sufficient to know the subsequence $\{D_{m0}\}$ $(m = 0, 1, 2, \cdots)$ in order to determine U since the remaining D_{mn} can then be determined (Cauchy's problem).

Received August 8, 1945. This paper was written while the author was associated with the Program of Advanced Instruction and Research in Mechanics at Brown University.