NON-SUMMABILITY OF THE CONJUGATE SERIES OF THE FOURIER SERIES

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1. **Introduction.** The conjugate series of the Fourier series generated by a function $f(x)$ which is Lebesgue integrable on $(-\pi, \pi)$ and of period 2π is, in the usual notation,

$$\sum_{n=1}^{\infty} (-b_n \cos nx + a_n \sin nx) \tag{1.1}$$

and has associated with it as "sum" the conjugate function defined as a Cauchy integral by

$$g(x) = -\frac{1}{2\pi} \lim_{\epsilon \to 0} \int_{\epsilon}^{\pi} \psi(u) \cot u/2 \, du, \tag{1.2}$$

where

$$\psi(u) = f(x + u) - f(x - u).$$

The set of values of x for which

$$\int_0^t | \psi(u) | \, du = O(t), \tag{1.3}$$

as $t \to 0$, is of importance in the theory of Cesàro summability of (1.1). It includes the values of x for which $\psi(u)$ is bounded for u sufficiently small, which is certainly true when $f(x)$ is bounded in the neighborhood of x.

Hardy and Littlewood [2; Theorem B], see also [1], show that if x_0 is a point for which (1.3) holds, then a necessary and sufficient condition for the Cesàro summability, (C, δ) with $\delta > 0$, of (1.1) is that the integral (1.2) should converge in some Cesàro sense; and that in particular, if $f(x)$ is bounded near the point x_0, it is necessary and sufficient that (1.2) converge in the ordinary sense. When (1.2) is definitely divergent Prasad [3] shows that (1.1) diverges in the same sense when summed by the Abel-Poisson method. He places no restrictions on $f(x)$ other than integrability and periodicity.

2. **Theorem.** In this note we prove the following theorem which may be considered as an extension of the results of Hardy and Littlewood, and Prasad given in §1.

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